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Integrating Safety in Developing a Variable Speed Limit System

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FINAL REPORT

Integrating Safety in Developing a Variable Speed Limits System

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LIST OF ACRONYMS/ABBREVIATIONS

Abbreviation	Full Name
AFR	Average Flow Ratio
AIC	Akaike’s Inform Criterion
ATM	Active Traffic Management
ATMS	Active Traffic Management System
AVI	Automatic Vehicle Identification
CDOT	Colorado Department of Transportation
CTM	Cell Transmission Model
DOT	Department of Transportation
DSL	Differential Speed Limits
ESS	Environmental Sensor Station
FCPI	Flow Crash Potential Indicator
GEH	Geoffey E.Heavers
LOS	Level of Service
MM	Mile Marker
MPC	Model Predictive Control
MLPNN	Multilayer Perceptron Neural Network
OAFR	Overall Average Flow Ratio
OD	Origin-Destination
PDF	Probability Density Function
PNN	Probabilistic Neural Network
RCI	Roadway Characteristic Inventory
RCRI	Rear-end Crash Collision Risk Index
RTMS	Remote Traffic Management Sensor
TT	Travel Time
VSL	Variable Speed Limits
VASL	Variable Advisory Speed Limits
VMSL	Variable Mandatory Speed Limits
VMS	Variable Message Sign

EXECUTIVE SUMMARY

Disaggregate safety studies benefit from the reliable surveillance systems which provide detailed real-time traffic and weather data. This information could help in capturing microlevel influences of the hazardous factors which might lead to a crash. The disaggregate traffic safety models, also called real-time crash risk evaluation models, can be used in monitoring crash hazardousness with the real-time field data fed in. One potential use of real-time crash risk evaluation models is to develop Variable Speed Limits (VSL) as a part of a freeway management system. Models have been developed to predict crash occurrence to proactively improve traffic safety and prevent crash occurrence.

In this study, disaggregate real-time crash risk evaluation models have been developed for the total crashes and the feasibility of utilizing a VSL system to improve traffic safety on freeways has been investigated.

This research was conducted based on data obtained from a 15-mile mountainous freeway section on I-70 in Colorado. The data contain historical crash data, roadway geometric characteristics, real-time weather data, and real-time traffic data. Real-time weather data were recorded by 6 weather stations installed along the freeway section, while the real-time traffic data were obtained from the Remote Traffic Microwave Sensor (RTMS) radars and Automatic Vechicle Identification (AVI) systems. Different datasets have been formulated from various data sources, and prepared for the multi-level traffic safety studies.

Real-time crash risk evaluation models have been developed to identify crash contributing factors at the disaggregate level. Support Vector Machine (SVM), a recently proposed statistical learning model and Hierarchical Bayesian logistic regression models were introduced to evaluate real-time crash risk. Classification and regression tree (CART) model has been developed to select the most important explanatory variables. Based on the variable selection results, Bayesian logistic regression models and SVM models with different kernel functions have been developed.

Substantial efforts have been dedicated to revealing the hazardous factors that affect crash occurrence from both the aggregate and disaggregate level in this study, however, findings and conclusions from these research work need to be transferred into applications for roadway design and freeway management. This study investigates the feasibility of utilizing Variable Speed Limits (VSL) system, one key part of ATM, to improve traffic safety on freeways. A proactive traffic safety improvement VSL control algorithm has been proposed. First, an extension of the traffic flow model METANET was employed to predict traffic flow while considering VSL's impacts on the flow-density diagram; a real-time crash risk evaluation model was then estimated for the purpose of quantifying crash risk; finally, the optimal VSL control strategies were achieved by employing an optimization technique of minimizing the total predicted crash risks along the VSL implementation area. Constraints were set up to limit the increase of the average travel time and differences between posted speed limits temporarily and spatially. The proposed VSL control strategy was tested for a mountainous freeway bottleneck area in the microscopic simulation software VISSIM. Safety impacts of the VSL system were quantified as crash risk improvements and speed homogeneity improvements. Moreover, three different driver compliance levels were modeled in VISSIM to monitor the sensitivity of VSL's safety impacts on driver compliance levels. Conclusions demonstrate that the proposed VSL system could effectively improve traffic safety by decreasing crash risk, enhancing speed homogeneity, and reducing travel time under both high and moderate driver compliance levels; while the VSL system does not have significant effects on traffic safety enhancement under the low compliance scenario. Future implementations of VSL control strategies and related research topics were also discussed.

CHAPTER 1: INTRODUCTION

1.1 Introduction

Active Traffic Management (ATM) is a scheme for improving traffic flow and reducing congestion on freeways (Mirshahi et al., 2007). ATM makes use of automatic systems and human interventions to manage traffic flow and ensure the safety of roadway users. This approach seeks to solve the congestion problems through mainline and ramp management strategies for freeway corridors. In addition, ATM is a tool that can maximize safety and throughput, which may be used as an interim strategy to maximize the efficiency of corridors that may ultimately receive major capital investments.

Among the ATM control strategies, Variable Speed Limit (VSL) systems have been widely used in the US and European countries. They represent a vital component of an Active Traffic Management System (ATMS), which has been suggested by FHWA as the next step in tackling the US freeway congestion problem (Mirshahi *et al.*, 2007). VSL systems have been employed nationwide on freeways to: (1) Reduce recurrent congestion; (2) Address adverse weather impacts on freeways; (3) Improve traffic safety; and (4) Improve air quality. Besides, speed limits can also be lowered when there is an incident or congestion on specific segments in order to reduce the chances of secondary accidents and facilitate a smoother flow of traffic (Sisiopiku, 2012).

VSL systems are beneficial for freeways as they can reduce speed variation and help maintain higher average speeds on freeways. This also leads to safer traffic conditions. In this study, we propose a freeway VSL control algorithm aims at improving traffic safety. The VSL system includes two main modules: (1) traffic flow analysis module and (2) crash risk assessment module. The traffic flow analysis module adopts an extension of the macroscopic traffic flow model METANET (Carlson *et al.*, 2010b) to analyze the effects of variable speed limits on traffic flow; while the crash risk assessment module utilizes a real-time crash risk assessment model to monitor the hazardousness of crash occurrence. With the purpose of minimizing crash risk over the controlled freeway section, optimal VSL control strategies are obtained through solving optimization problems. This study lies within the intersection of the Safety and Economic Competitiveness themes of the Center by maximizing the utility of existing resources through increased and more reliable freeway throughput while at the same time improving the safety performance of these facilities.

The effectiveness of the proposed VSL control algorithm has been tested for a mountainous freeway section on I-70 in Colorado through the micro-simulation software VISSIM (PTV, 2010). The chosen freeway section features of high crash rates, sharp horizontal curves, and longitudinal grades vary from 1.3% to 7% (absolute values). In addition, affected by the high altitudes, the climate (visibility, temperature, and precipitation) can vary abruptly within short distance. All these characteristics make this freeway section a challenging but interesting location for this VSL study. Five-minute average crash risk and speed standard deviation are then utilized as evaluation measures to assess the simulation results. Moreover, for the purpose of investigating sensitivities of VSL effects on driver compliance, three different levels of driver compliance are further coded, tested, and compared in the simulation software.

1.2 Outline of the Report

The remainder of this report is as follows. In Chapter 2 we present the summary of current implemented VSL systems in the US and Europe, which followed by recent research studies focused on advanced VSL control algorithms in Chapter 3. Chapter 4 describes the proposed VSL control algorithm, which contains the two main modules and the final optimization framework. Chapter 5 discusses the development and calibration of the simulation freeway section. Chapter 6 provides a detailed analysis of the simulation results with regarding to different driver compliance levels. Chapter 7 presents conclusions, discussions, and recommendations for the safety integrated VSL system.

CHAPTER 2: CURRENT IMPLEMENTED VSL SYSTEMS

In this chapter, we reviewed the implemented VSL systems in the US and Europe. VSL systems require the synergy of specific system objective(s), effective control algorithm, consistent evaluation, and improvement. VSL systems are operating in England, the Netherland, Germany, Finland, and Sweden; as well as in the US, e.g. Washington State, Minnesota, Missouri, and Wyoming. Five vital parts of the existing VSL systems are being discussed here in detail and a summary of the key issues are provided.

2.1 Objectives of the VSL systems

The VSL systems implemented in Europe and the US vary in objectives. The question rises here as why VSL should be adopted on a specific freeway segment or when should freeway managers consider the employment of VSL system on the freeways. Generally speaking, VSL systems have been used to (1) reduce recurrent congestion, (2) improve traffic under adverse weather conditions, and (3) improve traffic safety, among other reasons.

2.1.1 Reduce recurrent congestion

Recurrent congestion causes traffic delays and it is likely to increase the likelihood of crash occurrence. Several Variable Speed Limits (VSL) systems have been designed to eliminate or reduce the effects of recurrent congestion (or shockwaves). England operates their pilot 'Controlled Motorway' on M25 to reduce recurrent congestion. The objective of the pilot project was to reduce accident occurrence through controlling the traffic speed at peak hours (Robinson, 2000). The system attempts to predict when flow breakdown is about to happen, and introduces lower speed limits before that critical point is reached with the purpose of delaying its occurrence (Harbord, 1998). An operating Variable Advisory Speed Limit (VASL) system in the Twin Cities, Minnesota is being used to prevent the rapid propagation of shockwaves (Kwon *et al.*, 2011). Another VSL system aimed at solving the congestion problem was implemented on I-270, operated by the Missouri Department of Transportation (Bham *et al.*, 2010).

2.1.2 Address adverse weather conditions

Freeways suffering from adverse weather conditions have relatively high crash occurrence (Caliendo *et al.*, 2007;Malyshkina *et al.*, 2009;Yu *et al.*, 2013). VSL systems have been introduced to address the traffic safety problems caused by adverse weather conditions. Washington DOT employed VSL on I-90, which suffers rain and fog in summer and snow and ice in winter (Goodwin and Pisano, 2003). The road has a four times higher winter crash rate compared to the annual average; this is due to roadway geometry, truck percentage, and tourists that are unfamiliar with local conditions. In addition, a VSL system was implemented and operated on I-80 corridor in Southeastern Wyoming; the system was chosen to be implemented at this roadway because of adverse weather conditions during winter seasons and the more than 50 percent truck traffic flows (Layton and Young, 2011). Moreover, the Netherland also initialized speed harmonization (by utilizing VSL) on A16 only during adverse weather conditions.

2.1.3 Improve traffic safety

Traffic safety has always been one major concern of freeway management; several VSL systems have been implemented in the US and Europe with the only purpose of improving traffic safety. A study of a pilot project of the VSL system in Maryland has been documented in Pan *et al.* (2010). The chosen operated roadway segment was claimed to be suitable for VSL since: (1) Variety of geometric features, (2) High accident rate, and (3) High and dynamic traffic demand (Chang *et al.*, 2011). Germany has used speed harmonization since the 1970s, the system aimed at stabilizing traffic flow under heavy flow conditions, reducing crash probability, improving drivers' comfort, and reducing environmental impacts. Variable Speed Limits system has been set up at A8 between Salzburg and Munich, A3 between Sieburg and Cologne, and A5 near Karlsruhe; while Finland also utilizes VSL to improve speed harmonization to influence driving behavior and improve road safety (Mirshahi *et al.*, 2007).

2.1.4 Other objectives

In addition to the abovementioned three main motivations for implementing VSL systems, there are other objectives for the VSL system. Stoelhorst *et al.* (2011) documented a comprehensive program of field trials of VSL on motorways in the Netherlands between 2009 and 2010. The trials have been conducted at four locations, and the purpose varies as (1) shortening travel time (A2); (2) improving air quality (A12); (3) improving traffic safety and (4) increasing throughput.

2.2 VSL control algorithms - parameters used in the algorithms

VSL control algorithm plays a key role in the system; the implemented VSL systems mainly adopt rule-based control strategies which contain a set of thresholds and pre-set speed limits. Once the thresholds were reached, the corresponding speed limits would be adopted. However, since the VSL systems have distinct purposes, different parameters have been selected to be the thresholds. In the following part, according to the diversity of parameters, the existing VSL algorithms have been split into three groups.

2.2.1 Traffic flow parameters

Traffic flow variables (speed, volume, and density) are the most frequently utilized parameters in the control algorithms. The VSL control algorithm in Maryland has two modules: the first module computes the initial speed of each VSL location and the second module updates the displayed speed based on the difference between the detected traffic speed and target control speed (Chang *et al.*, 2011). Average speed and average travel time are calculated among all the detectors to decide the posted speed limits on the VSL signs (Pan *et al.*, 2010).

The advisory VSL system in Sweden uses two speed thresholds (V_{low} and V_{high}) to decide when to trigger the VSL system (Nissan and Koutsopoulosb, 2011). Average speed are captured from the overhead gantry detectors and aggregated into 5-minute intervals.

The UK motorways utilize dynamic and simple matrices control algorithm. When volume reaches 1,650 vehicles per hour per lane (vphpl), the speed limit is reduced from the default value of 70 mph to 60 mph. When volumes reach 2,050 vphpl, the speed limit is reduced to 50 mph. The control algorithm also includes speed information from the outstation, to detect queuing traffic. Once queues and slow moving traffic are detected, a speed limit of 40 mph is displayed immediately prior to the end of the queue (Harbord, 1998).

The posted speed limits on A2 in the Netherland are determined by a control algorithm based on 1-minute average speed and volume across all lanes. If an incident is detected, a speed limit of 50 km/h (31 mph) is displayed (Waller *et al.*, 2009). Besides, the VSL system located on New Jersey Turnpike also post speed limits based on the average travel speeds (McLawhorn, 2003).

Changes of speed limits in Missouri are determined by the lane occupancies or the time difference between how long vehicles used to pass a section of highway and how much time it would utilize under free flow condition. The speed limits could range from 60 mph during light traffic, to as low as 40 mph during extreme congestion (Bham et al., 2010).

The VASL system in Minnesota is designed to gradually reduce the speed levels of upstream incoming traffic when a bottleneck is found downstream (Kwon et al., 2011). The control algorithm first decides where the controlling zone should start, and then it identifies the speed limits according to the pre-set deceleration rate thresholds and the VASL control zone's length.

2.2.2 Weather and roadway surface condition variables

The VSL system in Wyoming adjust speed limits according to the recommended speed limits decided by Highway Patrol based on existing weather and roadway surface conditions (Layton and Young, 2011). While in the Netherlands, A16, the displayed speed limits are based on the visibility conditions captured by 20 visibility sensors along the road. If the visibility drops below 140 m (456 ft), then the speed limit will drop to 80 km/h (49 mph). If visibility drops below 70 m (228 ft), the speed limit will be dropped to 60 km/h (37 mph). Besides, if an incident is detected, 50 km/h (31 mph) on the first sign upstream and 70 km/h (43 mph) on second sign upstream will be displayed.

Finland's VSL algorithm only depends on the road conditions. Speed limits will be displayed as 120 km/h (74 mph) for good road conditions, 100 km/h (62 mph) for moderate road conditions, or 80 km/h (49 mph) for poor road conditions. The road conditions are decided by local weather (wind velocity and direction, air temperature, relative humidity, rain intensity, and cumulative precipitation) and road surface conditions (dry, wet, salted, and snowy).

2.2.3 Traffic and weather combined information

For a comprehensive VSL algorithm, traffic data and weather conditions are being considered together. For example, the control algorithm used in the I-90 VSL system reduces the speed limit in 10mph increments from 65 mph to 35 mph based upon prevailing road, weather, and traffic conditions (Goodwin and Pisano, 2003).

For German freeways, speed limits of 100, 80, or 60 km/h (62, 49, or 37 mph) would be displayed based on loop traffic data (volume and speed), environmental data measured by fog, ice, wind, and other detectors (Mirshahi *et al.*, 2007). Historic data are used to predict conditions over the next 30 minutes, and the volume of passenger cars and trucks speeds are also considered.

In the state of New Mexico, the speed limits are being changed according to the lighting and precipitation conditions. Displayed speed limits are calculated by the smoothed average speed plus an environmental constant defined by lighting and precipitating conditions, which varies from 30 to 55 mph (Robinson, 2000).

2.3 VSL Equipment

Equipments are the main part of an effective Intelligent Transportation System. While for the VSL systems, data collection devices and VSL displaying signs are the most important equipments. With the help of advanced data collection devices, more accurate traffic data along with real-time weather conditions can be considered in the control algorithms. As to the VSL displaying signs, with the technology moving forward, more messages can be delivered to the drivers from the control center though variable message signs (VMS) and changeable speed limit signs.

2.3.1 Data collection devices

The VSL system on I-35W (Minnesota) utilizes loop detectors to collect traffic data every 30 seconds (Kwon et al., 2011); microwave detectors were implemented on the gantries to measure traffic volumes and speeds in Sweden (Nissan and Koutsopoulosb, 2011). While in Finland, since the environmental variables are the only parameters in the algorithm, only data from weather stations are employed.

However, generally more than one type of data collection devices are utilized in the VSL systems: Road-side microwave traffic detectors and License Plate Recognition stations have been installed to capture the traffic flow parameters and travel time in Maryland (Pan et al., 2010); Washington Department of Transportation (DOT) implemented Environmental Sensor Stations (ESS), radar vehicle detectors, as well as digital radio, and microwave communication systems in their speed management system (Goodwin and Pisano, 2003).

2.3.2 VSL displaying devices

There are mainly two types of VSL displaying devices, overhead gantry signs and road-side signs. Road-side signs have been implemented on freeways in Maryland and Florida (Atkins, 2011). Overhead signs are more popular, it has been employed in the Minnesota, Wyoming, Washington, and VSL systems in the European countries.

Two different display technologies have been used in the Wyoming VSL system (Layton and Young, 2011), one is scrolling film technology and the other is LED display technology. The VSL signs were placed in pairs with one on the shoulder and the other on the median.

Driver behavior research has been conducted to evaluate the users' acceptance and compliance on different road facilities (Stoelhorst et al., 2011). The results indicate that VSL is best observed and understood when displayed on the matrix signs over each lane, instead of implementing them on the roadside as panels and signs.

2.3.3 VSL combined usage with VMS

In most cases, VSL signs are combined with Variable Message Sign (VMS) to provide more information to the drivers. In Maryland, VMS are employed to inform drivers of downstream traffic conditions and the estimated travel times (Pan et al. 2010). Virginia DOT (2011) installed a VMS prior to the VSL corridor to convey messages of whether VSL is in effect; and a static sign with "End Variable Speed Limit" had been placed before the exit point of the VSL corridor. For the New Jersey Turnpike, messages like "Reduce Speed Ahead" and the reason for speed limits reduction is displayed on VMS (Goodwin and Pisano, 2003).

While in England, the next generation of VMS is introduced to convey information about lane opening, closing, and speed controls to the drivers. Boice *et al.* (2006) evaluated the combination effects of VSL and traveler information systems provided by VMS on driver behaviors. The conclusions indicated that the most effectiveness situation is that drivers are warned of the approaching congested conditions, as well as the speed limit reduction.

2.4 Evaluation methods and results

The VSL system has become more and more popular because it can provide smoother traffic flow, safer traffic condition, and increased throughput during peak hours. Different measures have been employed in evaluation studies. Basically, traffic speed, travel time, and volume are the most commonly adopted evaluation criteria. However, besides the normal traffic flow parameters, some innovative ways of measuring the VSL system performance have also been created.

2.4.1 Traffic flow parameters evaluation

Simple traffic parameters such as average speed, traffic volume, and travel time have been widely used to evaluate the VSL systems:

- The average maximum deceleration rate, travel time, and peak hour lane volume are adopted to evaluate the VASL system in Minnesota (Kwon *et al.*, 2011). Less deceleration rate, reduced travel time, and higher peak hour lane volume have been achieved by the system in the 3 months' study period.
- Average speed and total volume through the roadway have been chosen as the evaluation parameters in the Pan *et al.* (2010) study. Results indicate that the VSL control system in Maryland provided a higher average speed and total volume during the 5 weeks' study period.
- Travel time and traffic speed were employed to evaluate the effects of VSL system under different scenarios (No control, Displayed estimated travel time, VSL control, and VSL combined with travel time display). Results indicate that the highway segments are being mostly beneficial under VSL&TT (Travel Time) display control, including travel time reduction and travel speed increment (Chang *et al.*, 2011).
- Average speed and the speed variance have been used in a study in Washington State. It was concluded with slightly speed variance increased and 13 percent reduced average speed for the speed management system (Goodwin and Pisano, 2003).
- Researchers in Finland focused on the effects of weather-controlled speed limits on the mean speed and average headways and traffic safety (Rämä, 1999). The study concludes that due to the lack of education to the drivers; most drivers thought the displayed speed limits as the recommended speeds rather than the maximum one. No significant benefits of VSL system have been detected.
- Heydecker and Addison (2011) investigate the relationship between speed and density to analyze the traffic flow under the operation of VSL. Good conclusions have been drawn from M25's traffic data assessment: the number of drivers exceeding the speed limit decreased by 50 percent and less lane changing was observed; in addition to a 15 percent increased flow on the slow lane. Furthermore, benefits also include an 18 percent

reduction in incidents, and money saved in millions each year on incidents and congestion.

2.4.2 Other evaluation methods

In addition to employ the traffic parameters as evaluation measures, a statistical way to measure the motorway capacity has been proposed in Nissan and Koutsopoulos (2011). Two distinct models based on traffic density have been established first by using average 5-minute traffic flow and speed data, then a generalized F-test was introduced to check the equalities of coefficients for the restricted and unrestricted models. Nevertheless, no significant differences of the flow-density have been detected before and after the VSL.

Geistefeldtl (2011) analyzed the effects of VSL on freeway capacities in Germany. A capacity function was introduced with traffic speed and density data utilized in the model. Instead of 1-hour capacity function, 5-minute intervals function was employed since it will provide more information about the traffic flow. Then the coefficient of variation of the estimated capacity distribution was used to compare 2- and 3-lane freeways with different speed control strategies. Results showed that freeways with VSL have significantly lower coefficient of variation compared with uncontrolled sections. Moreover, the traffic flow quality was evaluated by using Level of Service (LOS) under variety of control conditions (Nissan and Koutsopoulos, 2011). It was concluded that lower threshold of LOS E should be adjusted with different control conditions and this method will be implemented in the forthcoming edition of the German Highway Capacity Manual.

Duret *et al.* (2012) investigated the effects of VSL on lane flow distributions for a three-lane freeway in France. Results indicate that VSL affect the lane flow distributions and increase utilization of the shoulder lane by reducing the speed difference between the shoulder and the passing lanes.

Different evaluation standards were utilized in different trials in the Netherlands (Stoelhorst *et al.*, 2011). The number of days that over the limit value of PM_{10} was used to represent the environmental effect; the result shows that air quality has improved. Vehicle-delay hours per day were considered as the measure for throughput volume; while average speed was used as the traffic safety representative in rainy days. In short, better throughput, improved air quality, and enhanced traffic safety have been achieved by the VSL systems.

2.5 Driver compliance

The driver compliance issue is a vital factor for any freeway management strategies and it directly influences the effectiveness of the system. Two evaluation methods of speed compliance on the corridor were created in Layton and Young (2011). One strict standard determines the percentage of vehicles that are traveling at or below the posted speed limit, and the lenient one identifies the percentage of vehicles driving at not more than 5 mph above the speed limit. The authors concluded that trucks had a higher compliance rate than passenger cars since the total vehicles had the compliance range from 13% to 27% while trucks have a 57% compliance rate.

Turner *et al.* (2011) developed a model to calculate the compliance rate with the Variable Mandatory Speed Limits (VMSL) for the English Managed Motorways. The model assumes vehicle speeds follow a normal distribution with the mean being the average speed and the variance is related to the traffic demand at that time. Then the non-compliant vehicles are estimated using binomial experiments. By adopting the above mentioned model, the authors calculated the number of non-compliant vehicles in a dynamic way. The model was established using data from M42, and modified and tested by data from motorways under different traffic control strategies (hard shoulder running, fixed speed limit, and variable speed limits). Validations under various scenarios showed that the models maintain a less than 2% average error to predict the number of non-compliant vehicles. Moreover, the model keeps a balanced estimate for the freeways under different management strategies.

2.6 Overview

Key issues such as VSL systems' objectives and control algorithms have been described above, however, two summary tables (Table 2-1 and 2-2) are provided here to summarize these findings. Table 2-1 mainly sums up the VSL systems' location, objectives, and algorithm parameters. One point need to be mentioned is the system's regulation issue, several states and countries utilize VSL as advisory speed to drivers while others employed it as mandatory maximum speed limit. Systems in the Netherland adopt a red circle to indicate whether the speed limits are mandatory (Waller *et al.*, 2009). Table 2-2 mostly concludes the VSL devices, evaluation methods and results. Extra information in this table is the types (roadside or overhead) of the VSL signs: generally speaking, recently installed VSL systems utilize the overhead gantries to display the VSL along with other travel information while the relatively old systems employed simple and less costly roadside signs.

Table 2-1 Summarization of systems' regulation, objectives and algorithm parameters

VSL system location	Regulation	Objectives	Parameters used in the algorithms
MD 100, Maryland	Advisory	Reduce recurrent congestion	Average speed and travel time(1-min)
I-35W, Minnesota	Advisory	Prevent the propagation of the shock waves	Deceleration rate (30 seconds)
E4, Sweden	Advisory	Improve throughput	Average speed (5-min)
E18, Finland	Advisory	Harmonization speed	Roadway conditions
I-270, Missouri	Advisory	Solve congestion problem	Occupancy
I-80 in Wyoming	Regulatory	Adverse weather conditions	Weather and road conditions (5-min)
I-5,I-90, Washington	Regulatory	Adverse weather conditions	Weather, road and traffic conditions
I-40, New Mexico	Regulatory	Winter weather and road conditions	Lighting and precipitation conditions
M25, UK	Regulatory	Reduce recurrent congestion	Traffic volume
A3,A5 and A8, Germany	Regulatory	Stabilize traffic flow	Traffic data and environmental data
A2, Netherland	Changeable	Homogenization of traffic speeds and decrease travel time	Average speed and volume (1-min)
A16, Netherland	Changeable	Adverse weather conditions	Visibility

Table 2-2 Summarization of systems' devices, evaluation methods and results

VSL system location	Data collecting devices	Displaying devices	Evaluation methods	Evaluation results
MD 100, Maryland	Microwave detector and License Plate Recognition stations	Roadside signs	Average speed and total volume	Higher average speed and total volume
I-35W, Minnesota	Loop detector	Overhead signs	Average deceleration rate, travel time and peak hour volume	Less deceleration rate, reduced travel time and higher volume
E4, Sweden	Microwave detector	Overhead signs	Flow-density relationship	No significant impact on traffic conditions
E18, Finland	Weather station	Roadside signs	Average speed and headways	No significant improvement
I-80, Wyoming	Microwave detector	Overhead signs	85^{th} percentile speeds and standard deviation of speed(15-min)	No constant results have been achieved
I-5,I-90, Washington	Environmental Sensor Stations and radar vehicle detectors	Overhead signs	Average speed and speed variance	Reduced average speed
I-40, New Mexico	Loop detector	Roadside signs	Average speeds	Higher average speed
I-270, Missouri	Loop detector	Roadside signs	Volume and occupancy, average speed	Higher average speed, increased occupancy
M25, UK	Loop detectors and cameras	Overhead signs	Speed and density	Reduction in incidents and increased flow, less lane change
A3,A5 and A8, Germany	Loop detector	Overhead signs	Capacity function with speed and density	Lower coefficient of variance of speed
A2 and A16, Netherland	Loop detector	Overhead signs	Vehicle hours of delay per day	Better throughput
I-4, Florida	Traffic Sampling Stations	Roadside signs	Traffic flow and traffic safety	No significant impacts have been detected

CHAPTER 3: ADVANCED VSL CONTROL ALGORITHMS

In addition to the implemented systems we discussed last chapter, many research studies have been conducted to investigate advanced VSL control algorithms. These VSL research studies mainly oriented from two aspects: traffic safety improvement and freeway operation enhancement.

3.1 Safety improvement via VSL

3.1.1 Real-time crash risk evaluation analysis

VSL systems focused on safety improvement generally adopt real-time crash risk evaluation models to quantify crash hazardous. Oh *et al.* (2001) firstly proposed the novel approach to classify traffic conditions leading to a crash from real-time traffic data. This study utilized loop detector traffic data and historic crash data on a 9.2-mile freeway section on I-880 in Hayward, California. The authors proposed two distinct traffic conditions, which defined as normal traffic conditions and disruptive traffic conditions. The traffic data were aggregated into 5-minute intervals; and it was proved by t-test that the most significant variable for categorize the two traffic conditions is the standard deviation of speed. By utilizing the non-parametric density functions with kernel smoothing techniques, distributions for the standard deviation of speed have been achieved. Finally, a real-time application had been tested based on the Bayesian classification results. The proposed system showed potential effects in reducing crash occurrence likelihood and increasing safety.

Abdel-Aty et al. (2004) employed the matched case-control logistic regression modeling technique to predict freeway crashes based upon loop detector data. The study was focused on a 39-mile freeway section on I-4 in Orlando, Florida. 30-second loop detector data were aggregated into 5-minute interval to be utilized in the modeling procedure. The matched case-control logistic regression model was introduced to predict crash potentials. It was found that the coefficient of variation of speed at the downstream station and the average occupancy of upstream station are significant in the final models. The classification accuracies are more than 69% of crashes for the 1:5 matched dataset. Therefore, stations and significant variables with high hazard ratios have been selected. And it was decided to employ the logcvs at least 10-15 minutes prior to the crash as input in the classification models. 66% of the original data was used as the training dataset and the other used as evaluation dataset. Bayesian classifier methodology and probabilistic neural network (PNN) were adopted to classify the crash and non-crash cases. Different combinations of loop detector stations and numbers of loop detectors were tested; the results showed that with data from the crash station and two more upstream stations, the model can identified at least 70% crashes on the evaluation dataset.

Oh et al. (2006) proposed a surrogate method to evaluate rear-end crash risks. A rear-end crash potential index was created by assuming that, under the car-following situation, the stopping distances for the leading vehicle should be larger than the following vehicles'. With the advanced surveillance loop detectors, information about vehicle length, degree of symmetry, maximum magnitude, and vehicle types can be extracted. Moreover, RCRI (rear-end crash collision risk index) was calculated with the 5-minute level data and the fuzzy c-means algorithm was employed to cluster and stratified the index. With the real-time traffic data feed in, the authors claimed that the proposed system can be implemented in-field to monitor the traffic status to alert the potential rear-end crashes.

Pande and Abdel-Aty (2006b) developed a disaggregate traffic safety analysis model for the rear-end crashes on I-4 (Orlando area). The model was based on loop-detectors data at 5-minute aggregated level. Firstly, the authors utilized Kohonen vector quantization technique to cluster the crash data into two segments based on the average speed. After the clustering, classification trees were employed to formulate the rules to group the data into two segments. Moreover, variable selection technique has been performed to select most significant variables (traffic flow parameters and geometric design parameters) for both the two segments. Finally, multi-layer perceptron and normalized radial basis function neural networks have been chosen as the classification models. The results indicate that models can indentify 75% rear-end crashes with a 33% false alarm rate.

Lee et al. (2006) investigated the potential real-time indicators for sideswipe crashes on I-4 in Orlando area. The authors firstly calculated average flow ratio (AFR) for each specific lane and then they proposed a modified expression for the overall average flow ratio (OAFR) by considering: 1) the target lane of lane change is more important and 2) a geometric mean of lane flow ratios to represent the total number of lane changes in all lanes. Data used to calculate the AFRs were extracted for the time period of 5-10 minutes prior to the crash time. Then the authors employed logistic regression to classify the sideswipe crashes from the rear-end crashes, and the final best model included the OAFR, variation in flow, and a dummy variable indicating the peak periods. Moreover, the authors recommended that the proposed models can be used to help designing the ATMS crash prediction system.

Pande and Abdel-Aty (2006a) focused on lane-change related crashes on I-4 at Orlando metropolitan area. By examining the crash reports, all sideswipe collisions and angle crashes occurred on the inner lanes were classified as lane-change related crashes. A data mining based approach has been employed in the study: using classification tree technique to select important significant variables and utilizing neural networks to analysis and classify the crash and non-crash cases. As selected by the classification trees, variables like average speeds upstream and downstream of the crash site, average differences between adjacent lane occupancies upstream of the crash site, along with the standard deviation of volume, and speed downstream have been chosen as inputs in the neural network models. One thing need to be mentioned is that flow ratio variables is not significant in this study unlike in (Lee et al., 2006a). Finally, a hybrid model has been evaluated with online traffic data and it was found that the false alarm rates are relatively high, which could cause too many crash warnings to the drivers.

Abdel-Aty and Pemmanaboina (2006) incorporated weather information along with real-time traffic data to predict crashes on I-4 in Orlando, Florida. Loop detectors data were used to achieve real-time traffic data while five weather stations located at three airports nearby the study area were employed to achieve hourly rainfall information. Firstly, the authors used rainfall information from the five stations as independent variables and the actual crash report weather information as target variable to develop a logistic regression model for the rain index. Then, crash prediction models were performed with and without the rain index. Comparisons of the model fits indicated that the crash prediction model including the rain index outperformed the other one. Finally, the authors stated that the model can be used to calculate the probability of observing a crash versus not with online traffic and rainfall data.

Abdel-Aty et al. (2007) presented their works of improving traffic safety on freeways with real-time intervention strategies and crash risk assessment models. Firstly, the authors developed two separate models by splitting the whole crash data into two datasets regarding to the five-minute average speed observed just before the crash times. With the logistic regression modeling technique, models to predict crash potentials with traffic data prior to the crash times have been built. Then the authors employed a two-level nested logit model to analyze crash risks for the on-ramps and off-ramps. Based on the modeling results, the authors conducted a simulation based study using PARAMICS. Variable speed limits and Ramp Metering were utilized to reduce the crash risks for the high-speed situations and low-speed situations, respectively. For the experiments for the VSL system, variables like the pattern of speed limit change, the amount of change, the location of the change, the length of the speed limit change and the gap distance between the speed limit changes were tested. In the meanwhile, for the Ramp Metering simulations, the cycle length, green time per cycle, and the number of ramps that were to be metered have been determined. Results indicated that the proposed ITS strategies can effectively improve traffic safety situations on the freeway mainlines.

Pande et al. (2011) investigated the transferability issue for the real-time crash prediction models for the freeways in the Central Florida area. The authors utilized I-4 eastbound dataset to build the crash prediction models and then tested these models with data from westbound of I-4, I-95 northbound and I-95 southbound. All the studied freeway sections were equipped with traffic detectors (either loop detectors or radar detectors) collecting 30-seconds traffic flow conditions. In order to build the crash prediction models, the authors aggregated the raw data into 5-minute interval to avoid the noisy data, and then employed Random Forest technique to select the final input variables for the models. Multilayer Perceptron Neural Network (MLPNN) models were trained with the training dataset with different numbers of hidden neurons. Based on the prediction accuracy, MLPNN models with data from 4-station and 4 hidden neurons outperformed the other models; it was used to evaluate the transferability issues later on. Results indicated that the proposed model was able to perform on both the directions on I-4, however, it could not provide reasonable results for I-95 both directions. After investigating the basic traffic flow variables from the four studied areas, the authors concluded that the two corridors differ from driver population and travel patterns which resulted in an un-transferable crash prediction model.

Ahmed et al. (2012a) utilized AVI data along with real-time weather information and roadway geometric characteristics to formulate a real-time crash occurrence model. Logistic regression was performed with Bayesian inference technique. The finalized model showed that geometric factors are significant in both dry and snow seasons, while the 6-minute average speeds captured by the AVI system during the 6-12 minutes interval prior to the crash time and the 1-hour visibility before the crash time were also found to be significant in both seasons. Furthermore, specifically for the snow season, the 10-minute precipitation prior to the crash time was also significant. Results indicated that different active traffic management strategies should be adopted for the two distinct seasons.

Moreover, Ahmed and Abdel-Aty (2012b) investigated the viability of using automatic vehicle identification (AVI) data for real-time crash prediction for three expressways in Orlando area. The AVI data, frequently only used to estimate travel time between toll plazas, were employed to develop real-time crash prediction models. AVI data prior to the crash report time were aggregated into 5-minute level and totally 105 variables were invented to be analyzed. Random forest was adopted to select the most important variables that attribute to the crash occurrence. Later on, the authors developed matched case control logistic regression models to classify the crash and non-crash cases. Models have been estimated for the whole system and also for each specific expressway individually. Results of the models demonstrated a promising use of AVI data in predicting crashes on the expressways, if the AVI segments lengths are around 1.5 miles on average.

3.1.2 Detailed control strategies

This section discusses VSL studies from the traffic safety aspect; these VSL systems are designed to improve traffic safety and reduce crash occurrence. A primary element of a proactive traffic management strategy is model(s) that can separate 'crash prone' conditions from 'normal' traffic conditions in real-time. VSL systems focused on traffic safety usually include a function to quantify crash risk; here we call it crash prediction model. These crash prediction models were developed to quantify crash occurrence hazardousness and are used to decide when to trigger VSL control and evaluate VSL performance on traffic safety.

Real-time crash prediction models were estimated with the purpose of unveiling and identifying the crash precursors. With the advanced traffic surveillance system (loop detectors, speed radars, and automatic vehicle identification systems), traffic statuses prior to crash occurrence (usual 5-10 minutes prior to the crash time) would be identified for each crash; moreover, same data preparation procedures would be applied to randomly select non-crash cases. A dichotomous variable (1 represents crash cases while 0 indicates non-crash cases) is created to use as dependent variable; advanced statistical analysis models (logistic regression models (Abdel-Aty *et al.*, 2006a) and neural network models (Abdel-Aty *et al.*, 2008)) were employed to classify the crash and non-crash conditions. For example, for the logistic regression models, suppose the crash occurrence has the outcomes y=1 or y=0 with respective probability p and$1 - p$. Then the real-time crash prediction model can represent as:

$$logit\ (p) = \log(\frac{p}{1-p}) = \beta_0 + \boldsymbol{X\beta}$$

where β_0 is the intercept, $\boldsymbol{X}$ is the vector of the explanatory variables, $\boldsymbol{\beta}$ is the vector of coefficients for the explanatory variables. For the explanatory variables, statistical significant traffic flow characteristics (speed, volume and occupancy) were selected as input. Furthermore, based on the above equation, probabilities of crash occurrence can be calculated with the real-time traffic data.

VSL systems were set to be triggered when preset thresholds of the crash risks were reached: Lee *et al.* (2006) suggested four levles of threshold values of crash potential for the merging/diverging roadway sections and straight roadway sections spearately. As the threshold values increased, the intervention of VSL system is less frequently undertaken; Abdel-Aty *et al.* (2009) used speed difference between the upsteam average speed and average speed of VSL station of interest as the measure of whether VSL need to be implement or not. A 7 mph was used as a significant speed difference indicator as concluded in Cunningham (2007) that if the speed difference is larger than 7 mph, then the average rear-end crash risk would increase substantially.

After the VSL system takes control the traffic flow, crash risks were monitored in real-time. If the high crash occurrence risks have beem reduced with the lower speed limits, speed limits would gradually increase and go back to the base condition. Figure 3-1 shows the common procedures of VSL control strategies oriented from traffic safety approaches.

The evaluations of the VSL systems' effectives have concluded both the benefits from the traffic safety and freeway operation sides. Lee *et al.* (2006) quantified their VSL system with the overall crash potential; the control strategies were proved to be able to reduce the overall crash potential by 5-17%. Similarly, Abdel-Aty *et al.* (2006, 2008) visually showed the effects of VSL on traffic safety by plotting crash risk likelihood vs. simulation time and detector locations. Allaby *et al.* (2007) concluded that the modified VSL system was able to achieve safety improvements and less increased travel times.

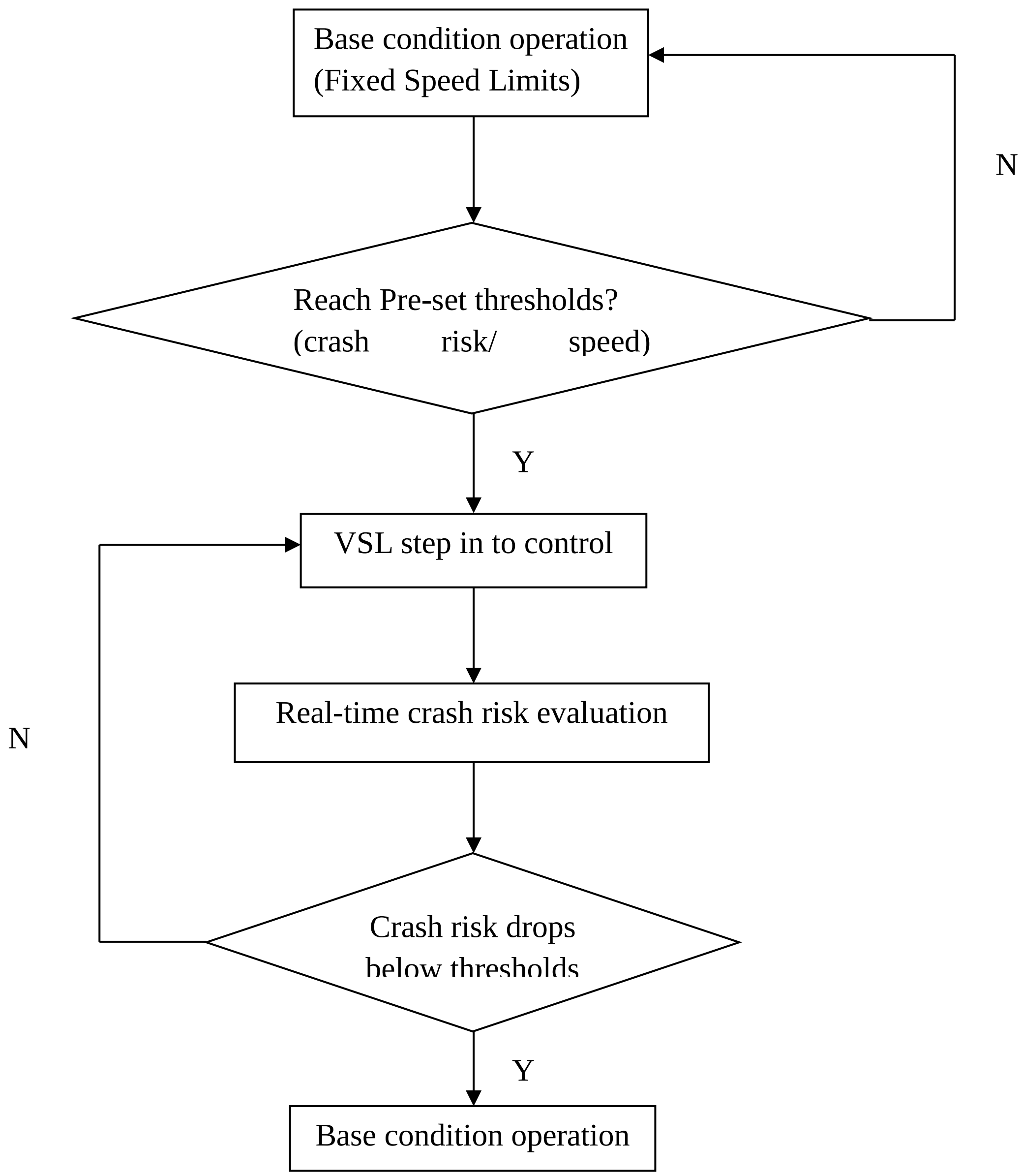

Figure 3-1 Flow chart of VSL control strategy oriented for traffic safety

Other than the commonly adopted control procedures shown in Figure 3-1, detailed control strategies were described in the following sections. Abdel-Aty *et al.* (2006) conducted a simulation based study on the variable speed limits effect on freeway safety improvement. A 36-mile freeway segment on I-4 that crosses Orlando downtown area was chosen for the study. The purpose of VSL was to improve its safety performance. Two crash prediction models were developed in a previous study (Abdel-Aty *et al.*, 2005) for two distinct traffic regimes (moderate-to-high-speed and low-speed traffic regimes). The simulation study was run in PARAMICS (Quadstone Limited, 2002), and this study focused on four key components of the VSL control strategies:(1) Speed change pattern (Abrupt or Gradual); (2) Upstream lowering and downstream raising distances; (3) Rate of change of speed limits (time step for change and speed step for change); and (4) Gap distance. After numerous test scenarios, the best control strategy was identified. The study concluded that VSL is effective for the high-speed regime while it seems to not have substantial crash risk reduction effect for the low-speed conditions. Moreover, comparisons of travel times between base case and VSL control cases showed a significant traffic time reduction with VSL.

In a latter study, Abdel-Aty *et al.* (2008) investigated using VSL to reduce rear-end and sideswipe crash risks on I-4 in Orlando. Unlike the previous work, a dynamic distance for the VSL implementation area has been considered and introduced in this study. The concept of homogeneous speed zones was created by comparing the speed differences between two contiguous segments and VSL would be effective based on the homogenous speed zone areas. Several important factors in the VSL implementation strategies have been addressed in the simulation study: (1) The speed limits decrease amount for the upstream speed limits (5 mph or 10 mph); (2) The need of simultaneous increase downstream speed limits; (3) The thresholds for homogenous speed zones (5 mph or 2.5 mph); (4) VSL control area (speed zone or half of the speed zone); and (5) the time periods for VSL implementations (5 or 10 min). Effectiveness for the proposed VSL control strategies were evaluated by comparing average crash risks to the basic control case. Results of the study concluded that VSL could successfully reduce the rear-end and sideswipe crash risks at low-volume traffic conditions while the system would have no benefits for the congested traffic conditions. Moreover, plotting the crash risks vs. locations can detect the potential crash risk migration effects, which was investigated by Abdel-Aty *et al.*(2006)

and concluded that lowering of crash risk at one location may be coupled with an increase in the crash risk at another location.

Lee *et al.* (2006) also developed a crash prediction model based on traffic flow characteristics and road geometric information to decide when to trigger VSL. A log-linear model was adopted based on crash and traffic data collected from a 10-km segment of the Gardiner Expressway in Toronto, Canada. Traffic speed and volume were imported from loop detectors into PARAMICS and the previous established crash prediction model was used to calculate the real-time crash potentials. Once the estimated crash potential exceeds a pre-specified value, speed limits are changed. In the simulation study, three major control strategies components have been analyzed: (1) thresholds of crash potential (conditions to trigger VSL control); (2) types of changing speed limit (increase or decrease the speed limits and by what amount), (3) durations of VSL intervention. After testing different scenarios, the authors claimed that 5-minute changing interval and 70 km/h speed limit is the best case. Base case and cases under variable speed limits have been compared and results illustrated that the variable speed limit can reduce the overall crash potential by 5-17% with a minor increase of total travel time.

Allaby *et al.* (2007) tested Variable Speed Limit sign system in PARAMICS based on simple tree logic control strategies. A threshold of occupancy was first chosen as the evaluation measure to trigger VSL and posted speed limits were decided by a tree logic model based on 20-s speed, volume and occupancy. Three speed zones (response zone, transition zone and temporal countdown) were defined to decide the displayed speeds for the upstream speed signs. However, after testing the base control algorithm, there were no promising results with both crash potential and travel time reduction. The authors made further improvement by testing various thresholds for triggering the VSL control and the tree logic model. After modifying the thresholds the author concluded that the VSLS system is able to provide safety improvements under heavily congested (peak period) and moderately congested (near-peak period) with no significant travel time penalty.

Kononov *et al.* (2012) proposed a potential VSL control algorithm oriented from traffic safety. A Flow Crash Potential Indicator (FCPI) was first introduced to reflect the crash probability for different operational regimes based on hourly volume, operating speed and free-flow speed. Then a critical FCPI was select to serve as a threshold to trigger the VSL system. The displayed speed limits can be calculated by the root mean of the critical FCPI divided by the observed flow density and rounded to the nearest 5 mph. Moreover, further improvements about this control algorithm have been proposed by achieving the critical FCPI for different traffic regimes (depending on AM/PM and weather conditions). However, no simulation work has been done to evaluate the proposed VSL system.

3.2 Traffic operation improvement via VSL

3.2.1 VSL impacts on traffic flow

Before discussing the detailed control strategies designed for traffic operation improvement, VSL's impacts on freeways' traffic flow have been investigated. Hegyi (2004) concluded that VSL's impact on the traffic flow diagram was merely replacing the left part of the flow-occupancy curve with a straight line; while Cremer (1979) concluded a quantitative model for the VSL-modified flow-occupancy diagram. The ratios stand for the applied VSL divided by the original non-VSL speed limits. Figure 3-1 shows the different influences concluded by the abovementioned studies (Carlson *et al.*, 2011).

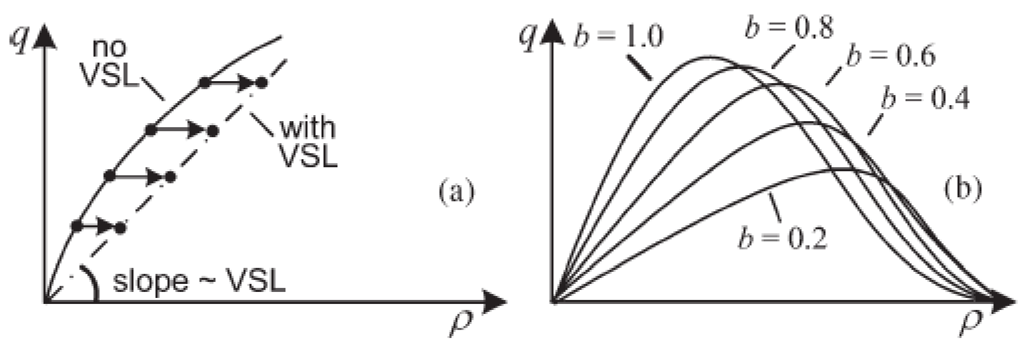

Figure 3-2(a) Hegyi (2004) model for VSL impact; (b) Cremer (1979) model for VSL impact

Later on, Papageorgiou *et al.* (2008) investigated the VSL impacts on aggregated traffic flow behavior (flow-occupancy diagram) by comparing the shapes of the flow-occupancy diagram under different speed limits. Effects of the VSLs on the diagrams have been investigated for under critical occupancy and cross-point of fitted curves with and without VSL scenarios. Results indicate that: (1) VSL introduces a visible slope decrease for the flow-occupancy diagram at under critical occupancies and (2) VSL strategies shift the critical occupancy to higher values in the flow-occupancy diagram. Based on the findings the authors proposed to use real-time estimated slopes of the flow-occupancy diagram to decide when to trigger the VSL. This is because the slope of the flow-occupancy diagram would approach zero when the flow approaches capacity, which is a good sign for triggering VSL.

Furthermore, Heydecker and Addison (2011) analyzed relationships between speed and density under the operation of variable speed limits. Traffic data were extracted from MIDAS system which contains flow, speed and occupancy information on a 188 km motorway in England. Data were aggregated over 1-minute intervals. The authors first defined the relationship between speed and density by using maximized log-likelihood values, and then a separate model has been built for each of the 4 lanes of traffic. The VSL system has four distinct posted values (70, 60, 50 and 40mph), after comparisons it was concluded that the capacity would increase when the speed control is introduced especially when the control speed is 50-60 mph.

3.2.2 Detailed control strategies

Researches for the VSL control strategies that stem from the traffic operation improvement are mainly designed to resolve the shock waves. Hegyi (2004) proposed to utilize speed limits to create a low density wave that propagates downstream, when the low-density wave meets the shock wave, it compensates its high density. VSL were designed to lower upstream speed limits of the bottleneck area which result in an increase of occupancy and decrease of average speed for the upstream area; these would delay the bottleneck activation and thus mitigate congestion.

Based on the theoretical rational, Hegyi *et al.* (2005) proposed an optimal coordination method to resolve the shock waves with the merit of Variable Speed Limits. The model predictive control (MPC) approach was introduced, which predicts the network evolution as a function of the current state and a given control input. The macroscopic traffic-flow model METANET (Messmer and Papageorgiou, 1990) was modified; the extensions of METANET model incorporated the speed limits as a parameter in the equations. The authors identified VSL control strategies by solving an optimization problem: the primary aim of the controller is to minimize the total travel time and several constraints like when the speed limits changed were selected. Both the continuous and discrete speed limits increments were tested and it turned out discrete speed limits can effectively reduce the total travel time by 17.3% with considering more safety-constraints than the continuous speed limits.

Later on, instead of the abovementioned algorithm which requires global optimization to find the speed limits, Popov *et al.* (2008) presented simplified controlling approach to resolve shock waves with distributed controllers. The studied freeway has been split into 20 segments with 0.5 km each and the central 10 segments were designed to test the VSL effects. Each segment would be assigned with a control whereas each controller using only the local information. Multi-objective evolutionary algorithms were employed to optimize the total travel time functions for each specific freeway segment. Different control structures have been tested in METANET– from using only local information to utilizing information up to 5 upstream segments and up to 1 downstream segment. Results showed by using only immediate neighbor information the VSL system can resolve the shock waves and reduce the total travel time by 20% compared to the basic uncontrolled case. This control strategy is easier for implementation since no high computational capabilities are needed.

Hegyi and Hoogendoorn (2010) proposed SPECIALIST algorithm to resolve shockwaves on freeways. The SPECIALIST is a simplified VSL control strategy which consists of four steps: shock wave detection, control scheme generation, resolvability assessment and control scheme application. After the shock wave was detected, lower speed limits would be assigned to control input volumes from upstream. According to the shock wave theory, six points on the flow-occupancy diagram were chosen to represent how the shock waves would be resolved. Moreover, the algorithm has been tested in the real field with a 14 km freeway section on the Dutch A12. Evaluations of the VSL control algorithm indicated that 80% of the shock waves that were theoretically resolvable (shockwave patterns are the same with the theoretical analyzed) were resolved in practice and totally 35 veh-hours were saved during the testing period. However, the SPECIALIST algorithm addresses the moving shock waves instead of the bottlenecks and is a feedforward scheme which may not be useable for other scenarios.

Carlson *et al.* (2011) investigated a local feedback based VSL control algorithm. Firstly, with similar approaches as Hegyi (2005) did; traffic demand prediction model was modified with the METANET model and then by optimizing the sophisticated model to find the best displayed speed limits. Considering the cumbersome model need extensive calculation and is not suitable for real-field implementation, the author proposed a cascade structure local feedback model. The novel control strategy is simple to implement in the field with one input (VSL rates, which represents different pre-specified displayed speed limits) and one output (the bottleneck occupancy). Various local feedback control models with different constraints have been tested and compared to the optimal control model which served as the upper limit of the achievable performance for the simpler feedback models. Results indicated around 15% travel time reduction can be achieved by the feedback control strategies; moreover, the improvements provided by the local feedback models are close to the optimal control model which showed promising results for real-field implementation with the simpler feedback models.

Wang and Ioannou (2011) proposed a dynamic VSL model with the consideration of driver behavior. The VSL model was designed as a car following mode and a speed limit tracking mode. Mostly the vehicles are running under the car following mode. However, the switch from car following mode to speed limit tracking mode only happens when the posted VSL is lower than the current speed and the vehicles can decelerate without violating safety considerations. The defined VSL model (three predefined VSL values -- 40km/h, 60 km/h and 80 km/h) was tested using microscopic simulation studies (VISSIM) and compared with a modified METANET model used in (Carlson *et al.*, 2010a). Both models were examined for a free flow condition and a 10-minute accident condition, and results demonstrated that the proposed model was more appropriate and effective than the METANET model.

Other than the METANET model, Cell Transmission Model (CTM) proposed by (Daganzo, 1994;Daganzo, 1997) has been modified to develop VSL control models. Lee *et al.* (2010) used the density as a main variable of VSL control module and applied the Demand-Supply method of Cell Transmission Model proposed by Daganzo (1997). VISSIM has been introduced to check the capacity and density changes under the influence of VSL. Average density, average travel time and total travel time were considered as the evaluation criteria, and the results showed that VSL control based on Cell Transmission method is effective in travel time reduction.

Moreover, Hadiuzzaman and Qiu (2012) developed a VSL control model with similar approach of Hegyi et al. (2005): by modifying the CTM model's fundamental diagram, a predictive traffic flow model was developed. Then the VSL control strategies were formulated as an optimization problem by minimizing the total travel time and total travel distance. The proposed VSL system is used to maximize bottleneck flow during peak hours since it was found that VSL is mostly effective during congestion periods. Simulation studies have been conducted in VISSIM with different scenarios and no-VSL control base case. Benefits from the VSL can be concluded as significant throughput increase and travel time reduction.

Driver behavior influences on the VSL effects have also been considered and investigated. Nes *et al.* (2010) conducted a driving simulator study to assess different sophistication levels of a dynamic speed limit system, as well the homogeneity of driving speed and acceptance of the different dynamic speed limit systems were also tested. Forty six subjects completed the study, each subject had to drive 6 road segments with fixed speed limit at 80 km/h and 6 road segments with dynamic speed limit either 70 km/h for dangerous situations or 90 km/h for non-dangerous situations. Standard deviation of the average speed of a subject on a road section has been measured as the homogeneity of individual speeds, and standard deviation of the average speed for all subjects on a road section was used to represent the homogeneity in speed between subjects. ANOVA was introduced to determine the effect of the speed limit system, and it was concluded that (1) under the dynamic speed limits control, subjects showed more homogeneity of driving speeds, (2) the advanced in-car system has the highest homogeneity, (3) subjects were positive about accepting the dynamic speed limit system and (4) they gave more credit to the dynamic speed limit system than static speed limits.

3.3 Summary

Variable Speed Limit systems have great potential and effects in creating smoother, safer, and less congested freeways. Based on the review of the current practice VSL systems (Chapter 3) and advanced research studies (Chapter 4), suggestions and conclusions for future implementation and research directions are discussed below.

VSL control algorithm is the key component of the system; it must meet the desired system implementation purposes and also be efficient with less computation requirements. For each control strategy there are four major questions that need to be answered: (1) when to trigger the VSL; (2) how to change speed limits; (3) how many VSL stations need to be coordinated; and (4) when to go back to the base condition. For the implemented VSL systems, majority of them adopted simple rule based control strategies, where speed limits would be changed based on pre-specified thresholds of traffic flow and weather information.

In addition, more advanced control strategies have been tested in research studies. These studies mainly stem from two purposes: improve traffic safety and reduce traffic congestion. Studies focused on traffic safety improvement introduced a crash prediction model to evaluate crash risks in real-time; and other detailed control strategies were decided by well designed multi-times simulation runs. For the studies that meant for reducing traffic congestion, model predictive control approach has been widely used. Speed limits were incorporated into macroscopic traffic flow models (METANET or Cell Transmission Model); best control strategies were identified by solving an optimization problem, which minimize the total travel time and choose several safety consideration constraints.

Based on the reviews, several directions for future research can be proposed. First, newly implemented VSL systems adopt overhead gantries to display speed limits for each lane; differential speed limits (DSL) which now mostly used to display different speed limits for passenger cars and trucks can be used combined with VSL. The DSL display different speed limits for each lane, for example, higher speed limits for left lanes with most passenger cars while lower speed limits for right lanes with trucks running on them. Although the DSL systems might increase the speed variance between lanes, with real-time changeable speed limits, the combined usage with VSL should be very attractive.

Second, crash migration caused by the VSL needs to be further investigated. As most VSL studies that stem from traffic safety have concluded a crash risk reduction for the locations where VSL were implemented, more crashes might occur at the upstream and downstream locations due to the migration. In the literature, only Abdel-Aty et al. (2008) checked this issue by plotting crash likelihood vs. locations. Therefore, more research is needed before concluding that VSL could improve traffic safety.

Third, only one improvement (traffic safety or operation) were achieved in the aforementioned studies. However, future studies may focus on the multi-objective VSL systems. With the MPC approach, predicted traffic flow parameters can be used as input to calculate predictive crash risks. Using the same optimization approach, the objectives can be set up to minimizing the crash risks and total travel times. However, how to balance these two objectives and how to improve the prediction accuracies needs more attention. Furthermore, existing MPC approaches used macroscopic traffic flow model, feasibility of utilizing microscopic model (e.g., car following models) to predict the upcoming traffic flow status should be tested.

In addition, the effects of different VSL displaying technologies on drivers' compliance can be investigated. Beside the speed limits, some messages like "queue ahead" or "speed enforced" can be displayed on Variable Message Signs (VMS) on the gantries or at the roadside. Messages displayed by VMS should be chosen with care as stated in Hassan and Abdel-Aty (2011) that driver's satisfaction with VSL and VMS is the most important variable that positively affects driver's compliance with VSL and VMS instructions under different fog and traffic conditions. Besides, the enforcement of VSL should be considered according to local driver acceptance and behavior features and incorporated into the display of VSLs. Finally, as the advanced methodologies would enhance VSL effects at different aspects, practitioners are always pursing a reliable, efficient, simpler, and local based VSL system.

CHAPTER 4: VSL CONTROL ALGORITHM

This chapter discusses the proposed VSL control algorithm. In order to develop a safety integrated VSL system, both the VSL effects on traffic flow and safety need to be considered during operation. The proposed VSL control algorithm contains two major modules: (1) traffic flow analysis module and (2) crash risk assessment module. The traffic flow analysis module utilize an extension of METANET traffic flow model to analyze potential effects of variable speed limits on traffic flow, which has been widely adopted in the previous studies focused on traffic flow improvement VSL systems (Carlson *et al.*, 2010a, 2010b). In addition, as a safety integrated VSL system, one main objective of the system is to reduce crash occurrence. The crash risk assessment module is developed to provide a real-time crash risk evaluation measurement. Finally, the best VSL control strategies are achieved by solving optimization problems of minimizing crash risk for the controlled freeway section. In this chapter, the two major modules in the control algorithm will be discussed in detail; and the final optimization approach is also illustrated.

4.1 Traffic flow analysis module

The traffic flow analysis module adopts an extension of METANET model to evaluate VSL effects on traffic flow. In the METANET model (Messmer and Papageorgiou, 1990), freeway sections are divided into segments where each segment has uniform characteristics in geometry. A freeway section m is split into N_m segments with length of L_m and λ_m lanes. Traffic flow in each segment at time instant $t = kT$ is depicted by the variables of traffic density $\rho_{m,i}(k)$ (veh/lane/mile), mean speed $v_{m,i}(k)$ (mph), and traffic volume $q_{m,i}k$ (veh/h); where T is the time step used for traffic flow prediction ($T = 5\ \text{min in this study}$). Traffic variables for each segment i of freeway section m are calculated through the following equations:

$$\rho_{m,i}(k+1) = \rho_{m,i}(k) + \frac{T}{L_m \lambda_m}\left[q_{m,i-1}(k) - q_{m,i}(k)\right]$$

$$q_{m,i}(k) = \rho_{m,i}(k) \cdot v_{m,i}(k) \cdot \lambda_m$$

$$v_{m,i}(k+1) = v_{m,i}(k) + \frac{T}{\tau}\Big(V\left(\rho_{m,i}(k)\right) - v_{m,i}(k)\Big) + \frac{T}{L_m} v_{m,i}(k)\left(v_{m,i-1}(k) - v_{m,i}(k)\right) - \frac{\eta T}{\tau L_m}\frac{\rho_{m,i+1}(k) - \rho_{m,i}(k)}{\rho_{m,i}(k) + \kappa}$$

$$V\left(\rho_{m,i}(k)\right) = v_{f,m} \cdot \exp\left[-\frac{1}{a_m}\left(\frac{\rho_{m,i}(k)}{\rho_{cr,m}}\right)^{a_m}\right]$$

where $v_{f,m}$ represents the free-flow speed of link m, $\rho_{cr,m}$ denotes the critical density per lane of section m, and a_m, τ, η, κ are constant parameters to be calculated.

As investigated by Carlson *et al.* (2010a) and used in Carlson *et al.* (2010b), a quantified model was used to illustrate VSL-modified flow-occupancy diagram. The VSL rates $b_m(k)$ stand for the ratios of applied changeable speed limits and original constant speed limits. Influences of VSL on the flow-density diagram can be quantified as:

$$v'_{f,m} = v_{f,m} \cdot b_m(k)$$

$$\rho'_{cr,m} = \rho_{cr,m} \cdot \{1 + A_m \cdot [1 - b_i(k)]\}$$

$$\alpha_m{}' = \alpha_m \cdot [E_m - (E_m - 1) \cdot b_i(k)]$$

where $v_{f,m}, \rho_{cr,m}, \alpha_m$ denote for the non-VSL values for the three parameters; A_m, E_m are constant parameters that represent VSL impacts on the fundamental diagram and they are to be estimated based on real data.

4.2 Crash risk assessment module

In the crash risk assessment module, we incorporate a crash risk evaluation model to evaluate crash hazardous in real-time. Traffic crashes are complex events which involve human and environmental hazardous factors, roadway geometry characteristics, and traffic flow conditions. Since micro-simulation software cannot reproduce crashes directly, a surrogate traffic safety measurement needs to be proposed to evaluate the safety improvements brought by VSL systems. Developing real-time crash risk evaluation models is the frequently adopted approach to quantify the hazard of crash occurrence in VSL simulation studies (Abdel-Aty *et al.*, 2006a;Lee *et al.*, 2006). The crash risk evaluation model in this study utilizes a logistic regression model to measure crash risk with historical crash data and real-time traffic data matched to each crash case.

Suppose crash occurrence has the outcome of $y = 1$ (crash cases) or $y = 0$ (non-crash cases) with the respective probability p and $1 - p$. The logistic regression can be explained as follows:

$$y \sim Binomial\ (p)$$

$$logit\ (p) = \log(\frac{p}{1-p}) = \beta_0 + \boldsymbol{X\beta}$$

where β_0 is the intercept, $\boldsymbol{X}$ is the vector of the explanatory variables, $\boldsymbol{\beta}$ is the vector of coefficients for the explanatory variables. In this study, the three traffic flow parameters from the METANET model (average speed, density, and traffic volume) were the candidate explanatory variables in the crash risk evaluation model. Since traffic flow parameters of the $k + 1$ time interval can be achieved from the METANET model, crash risks for $k + 1$ time interval can be calculated.

4.3 VSL optimization

The objective of this study is to utilize VSL system to improve traffic safety. Suppose roadway section m is divided into N_m links, therefore, the objective function for VSL optimization is set up to minimize the total crash risk for section m at time step $k + 1$:

Minimize $\sum_{i=1}^{N_m} CR_i\ (k+1) = \frac{e^{\beta_0 + \boldsymbol{X}(k+1)\boldsymbol{\beta}}}{1+e^{\beta_0 + \boldsymbol{X}(k+1)\boldsymbol{\beta}}}$

where $\boldsymbol{X}(k + 1)$ is the vector of traffic flow parameters provided by the extended METANET model. $b_i(k)$ represent the optimal VSL rates that should be implemented for segment i at time step k, where $b_i(k) \in [b_{min}, 1]$ as $b_{min} \in (0,1)$ is the lowest admissible bound for the VSL rates; $b_{min} = 0.65$ is used in this study to set the minimum speed limit at 40 mph (the 15 percentile speed under fixed speed limit is 42 mph; which was rounded to 40 mph to set the minimum speed limit).

Combining all the previously described equations, the optimization problem can be displayed as

$$b_i(k+1) = f[b_i(k), u(k)], \quad b_i(0) = 1$$

where b_i is the VSL rate of segment i in link m. $\boldsymbol{u}$ is the traffic flow state vector which contains inputs of the METANET model (speed, density, and volume information). From the equation it can be seen that inputs of this model are traffic flow parameters and current VSL rates, and the optimal VSL rates for next time step are the only outputs.

In addition, with the consideration of traffic operation and safety, constraints are set up for:

a) the maximum increase of average travel time for the VSL control area compared to the non-VSL control cases is 5 percent;
b) the maximum difference between two neighboring posted speed limits is 10 mph (spatial constraint);
c) the maximum difference between two consecutive VSL control time steps is 10 mph (temporal constraint).

The average travel time increment control can be formulated as:

$$\sum_{i=1}^{N_m} \frac{L_i}{v_i(k+1)} \leq (1+t_m) \sum_{i=1}^{N_m} \frac{L_i}{v_i{}'(k+1)}$$

where $v_i{}'(k+1)$ is the average speed under non-VSL control ($b_i'(k) = 1$); t_m is the average travel time increase rate, which is set as 0.05 in this study. Additionally, temporal and spatial constraints for the posted speed limits can be showed as:

$$|b_{i+1}(k) - b_i(k)| \leq 0.167 \; for \; i = 1, \dots, n-1 \; (60 * 0.167 \cong 10)$$

$$|b_i(k+1) - b_i(k)| \leq 0.167 \; for \; i = 1, \dots, n$$

CHAPTER 5: SIMULATION MODEL DEVELOPMENT

For the purpose of evaluating effects of the Variable Speed Limits (VSL) system, microscopic simulation software VISSIM, was selected to evaluate the designed traffic management strategies. Traffic simulation models have become more and more important and useful in Intelligent Transportation System related studies. Researchers and traffic engineers benefit from the low cost and zero hazardous evaluation tools for ITS systems like Active Traffic Management systems during the planning procedure. In order to properly represent the studied freeway section (geometric characteristics, traffic flow, and speed distribution), network coding, calibration, and validation work for the simulation network have been done sequentially. Chronologically, the major steps involved include:

1) Background Building
2) Network Coding (links, connectors, and detectors)
3) Network Calibration and Validation

5.1 Background building

In order to accurately establish the freeway section in VISSIM, background images with a large scale for the studied roadway section are required. A scale of 1:5000 was selected to view the freeway section in ArcMap (ESRI, 2006). Pictures with portions of the freeway section have been captured and saved for future network building procedure. As showed in Figure 11-1, screenshot of a part of the studied roadway section was captured from ArcMap. With a sufficiently large scale, detailed geometric characteristics (number of lanes, shapes of curvatures, and ramps features) can conveniently be detected from these screenshots. Moreover, for the easiness of locating in-field ITS devices (locations for the RTMS (Remote Traffic Microwave Sensor) detectors and speed limit signs), locations of the milepost (red dots in Figure 5-1), RTMS detectors (green dots), and potential VSL signs (triangles) were also pre-mapped in the ArcMap system; locations of these equipment were also captured and saved by the freeway screenshots. A total of 25 roadway segment screenshot images were extracted and archived for the 15-mile freeway section (from MM (Mile Marker) 205 to MM 220). Therefore, the 25 segment images were further combined into four files (Figure 5-2 - Figure 5-5) for the purpose of being loaded as background images into VISSIM; image combining work was done in Adobe Photoshop (Photoshop, 2000). After merging, four background images were carefully placed and appropriately scaled in the microscopic simulation software. Figure 5-6 shows the final freeway section's background images that have been loaded in VISSIM.

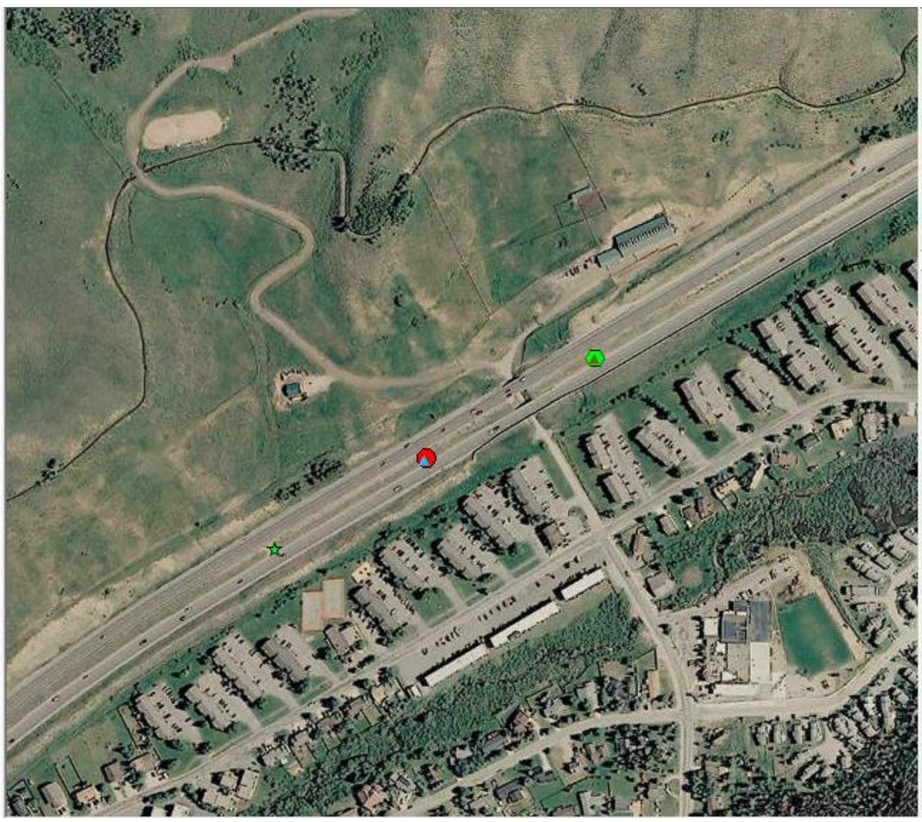

Figure 5-1 Roadway segment sample image captured from ArcMap

Figure 5-2 Background roadway segment image-1

Figure 5-3 Background roadway segment image-2

Figure 5-4 Background roadway segment image-3

Figure 5-5 Background roadway segment image-4

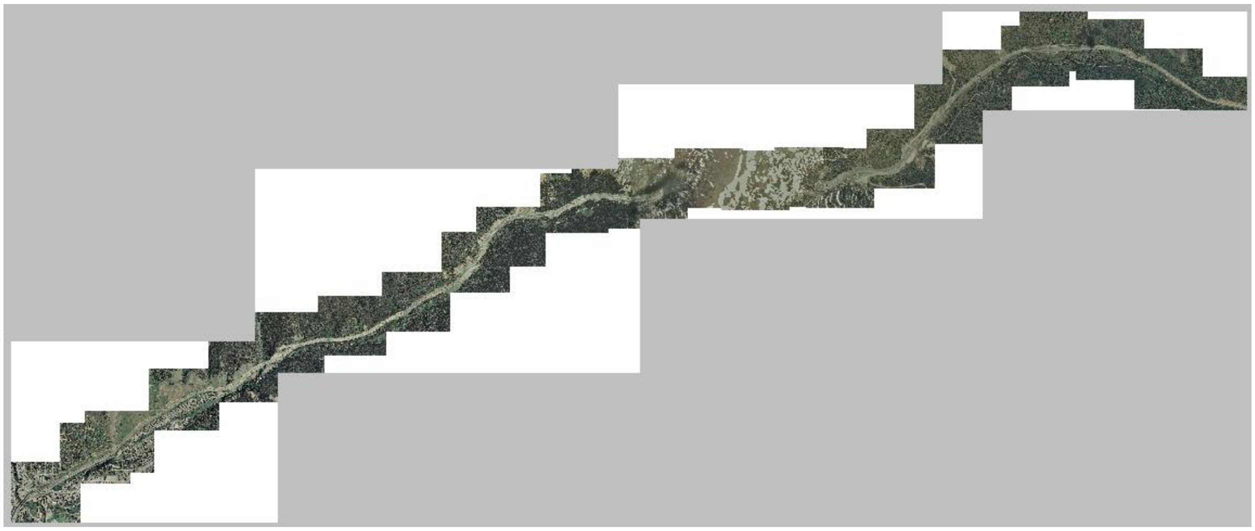

Figure 5-6 Background images in VISSIM

5.2 Network coding

Roadway networks in VISSIM consist of links and connectors. Links were used to represent single or multiple lane roadway segments, which have a specified direction of traffic flow. Connectors were used to connect two consecutive links; vehicles cannot continue in the simulation network if one link was placed on top of another link without a connector. For each link, several properties must be specified: (1) number of lanes for the segment; (2) behavior type (option three, freeway was selected in this study); (3) lane width; and (4) gradient information. Moreover, by activating the “Generate opposite direction” option, VISSIM would automatically generate another link for the opposite flow direction with similar configuration. However, due to the large variation of geometric characteristics of the studied freeway section, this function was not used. The East and West bounds of the freeway section were coded separately. Curvatures of the freeway were coded through adjusting the shapes of links and connectors to follow the roadway shapes in the background maps. Finally, the 15-mile freeway section has been carefully coded in VISSIM with 76 links and 73 connectors. Figure 5-7 displays a coded freeway segment with the background map and Figure 5-8 shows whole the coded freeway network; for better visibility links were represented as blue lines while the connectors were red lines. The coded freeway could accurately represent geometric characteristics of the studied freeway segment.

Figure 5-7 Coded freeway section with background image

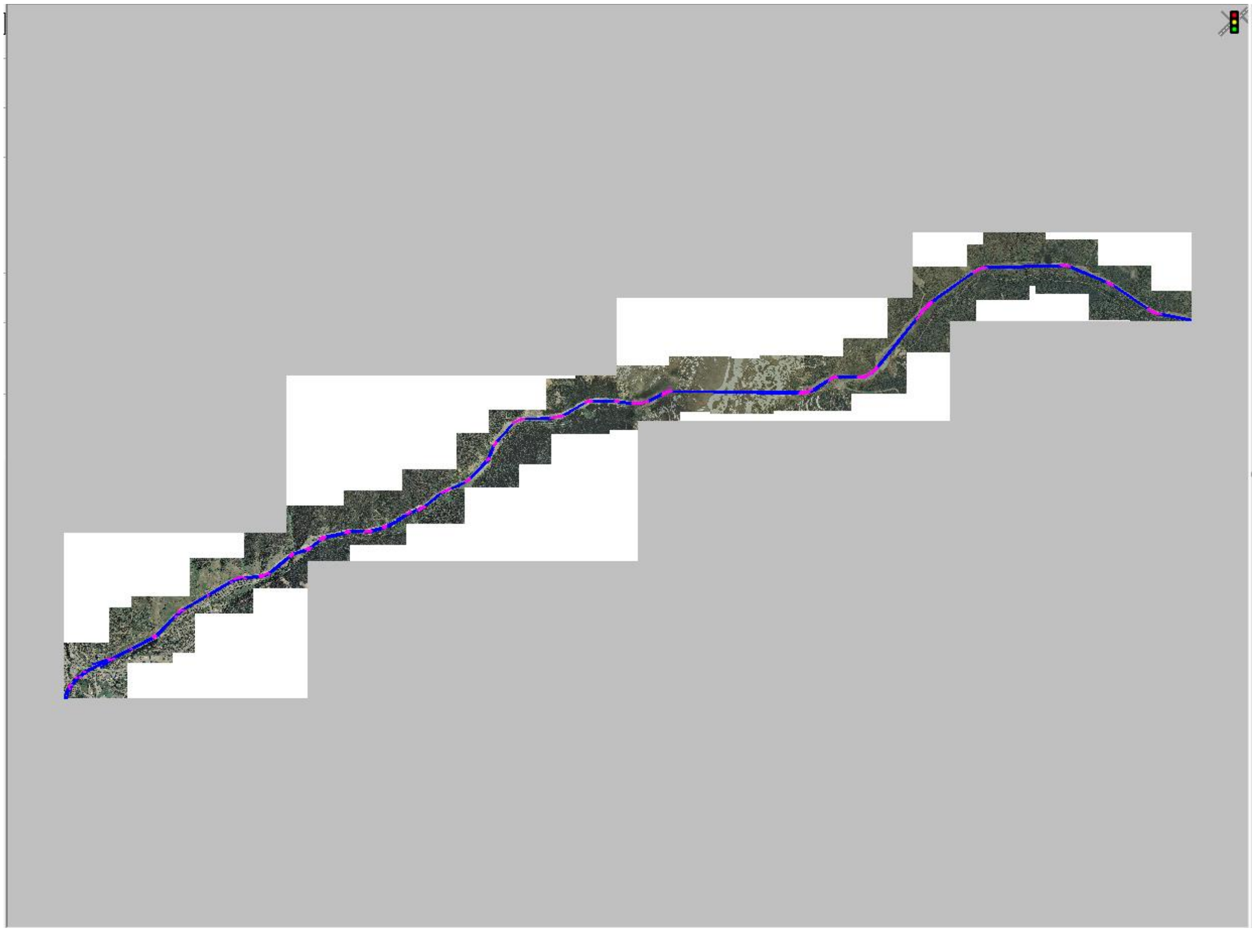

Figure 5-8 Coded freeway network with background image

In addition to the coded freeway section with geometric characteristics as the actual network, data collection points were added to the simulation network. Data collection points were installed at the exact locations of in-field RTMS radars, for the purpose of capturing simulated traffic flow parameters and further compared with in-field data. Traffic volume and speed information from the simulation models can be archived with the coded data collection points. However, one data collection point can only catch information for one single lane, multiple data collection points were placed at different lanes of the two-lane and three-lane roadway segments. Figure 5-9 shows an example of the coded data collection points. After matching all the data collection points with respect to the RTMS radar detectors' locations, the simulated network would contain the same geometry, traffic control operation, and configurations of ITS facilities as those in the actual field.

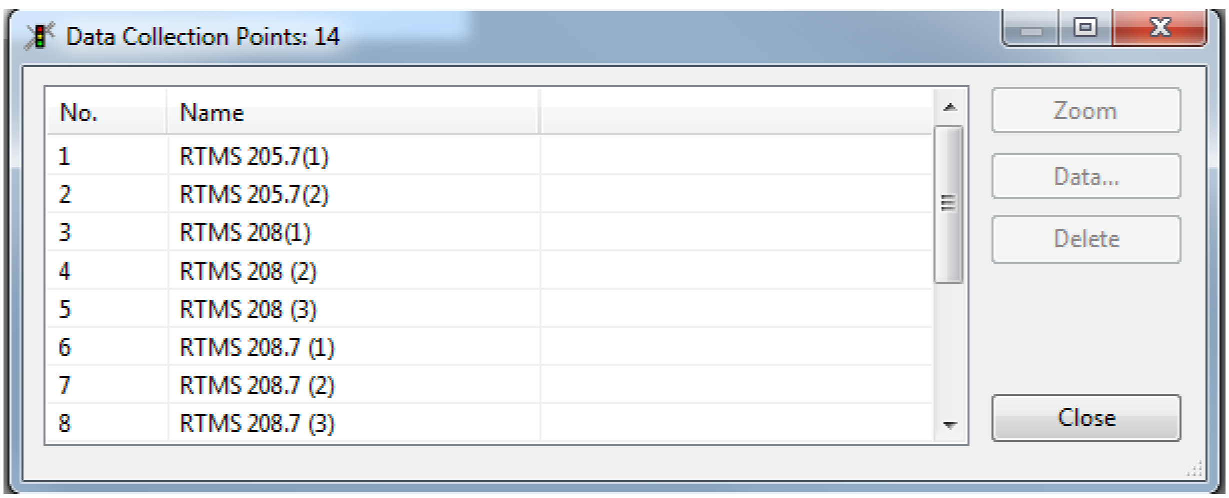

Figure 5-9 Data collection points defined in VISSIM

5.3 Network calibration and validation

After coding the studied freeway network in VISSIM, calibration and validation work are needed to reflect the in-field traffic characteristics in the simulation network. The simulation model of I-70 was calibrated for traffic volume at 5-min intervals and validated utilizing speed values in this study. The calibration and validation efforts require comparing the simulated traffic data with the observed in-field traffic data. Since traffic variables (e.g. traffic volume) vary from day to day, average values of traffic variables over a month were used in this study. The simulation model was calibrated and validated to re-construct the in-field traffic characteristic for morning peak hours (9 to 11 AM) of weekdays in August, 2011. The calibration and validation process used in this study is shown in Figure 5-10.

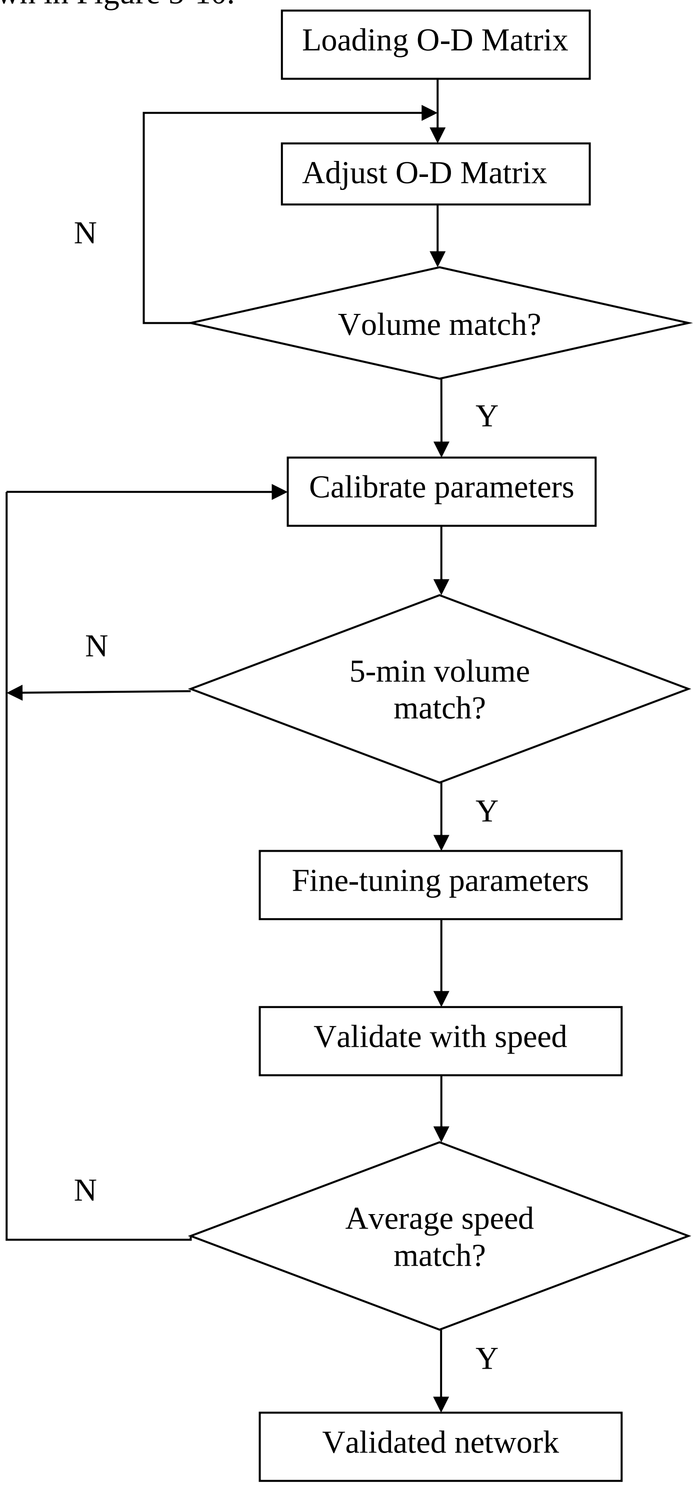

Figure 5-10 Flow chart of calibration and validation procedure

5.3.1 Preparation of calibration data

In order to calibrate the simulation network, in addition to the network geometric characteristic data which have already been coded in VISSIM, the following data are still needed:

1) Hourly volume data: average hourly volumes of weekdays (August, 2011) have been obtained from 26 radar detectors (13 radars per direction). Hourly volumes for the freeway mainline, on-ramps, and off-ramps were used to formulate the Origin-Destination (O-D) flow matrix (morning peak hours from 9 to 11 AM were chosen as the study time period).
2) 5-minute volume data: the RTMS data provide traffic counts at the 30-second interval, which were extracted from the database for the time period from Aug 1, 2011 to Aug 31, 2011. These 30-second raw data were further aggregated to the 5-minute interval and prepared for the network calibration procedure.
3) Vehicle composition data: to reflect the vehicle mix by type, truck percentages in the traffic flow were obtained from Roadway Characteristic Inventory (RCI).
4) Speed distribution data: speed values detected by RTMS radars were utilized to formulate the cumulative speed distribution.

5.3.2 Network calibration

The VISSIM simulation model of I-70 was calibrated for volume and validated with speed values. An origin-destination (O-D) flow matrix (containing mainline volume, on-ramp entrance flows, and off-ramp exit flows) has been obtained through the RTMS data. One thing noteworthy is that, RTMS radars only provide flow information for mainline segments, no ramp volumes were available. Nevertheless, as a restricted entrance freeway, ramp volumes have been acquired through comparing volume values from detectors located in upstream and downstream areas of ramps.

After obtaining the O-D flow matrix, vehicle composition information was defined in VISSIM. According to the Roadway Characteristic Inventory (RCI), mean truck percentage for the freeway section is 10.17%. Two types of vehicles (Car and HGV (truck)) were inputted into the simulation network; the relative flows were set to 0.89 and 0.11, respectively. Figure 5-11 shows the VISSIM settings for the vehicle composition.

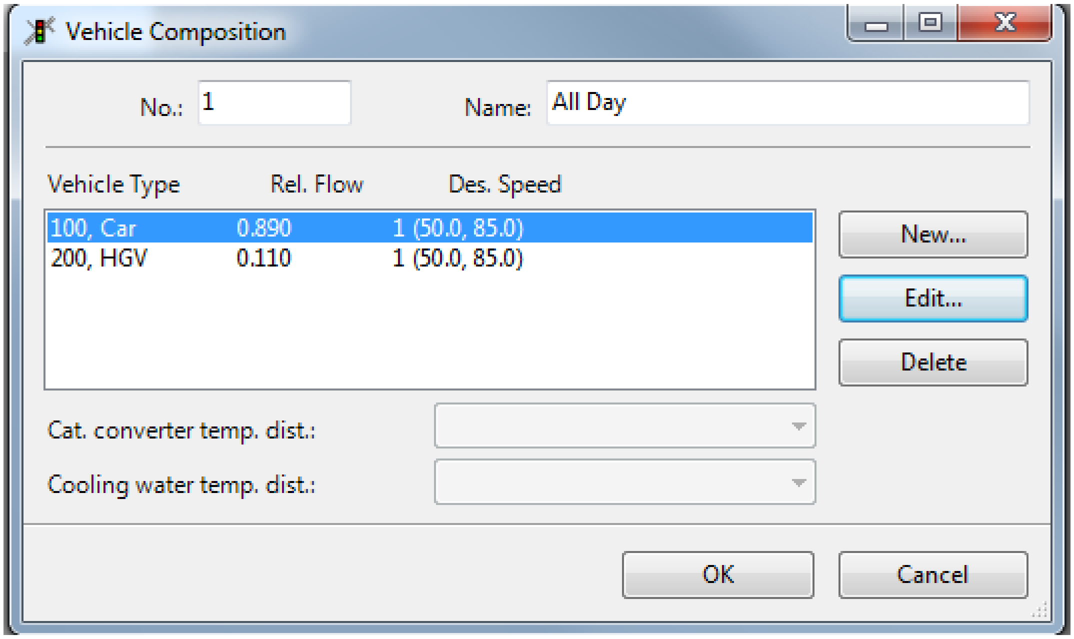

Figure 5-11 Vehicle composition for the freeway section

Besides the O-D matrix and vehicle composition information, desired speed distribution data are required to be acquired from in-field speed data. Cumulative speed distributions were formulated with the RTMS speed data; Figure 5-12 displays the speed distribution yield from in-field data (speed limit 60 mph). Through matching the intermediate points' values, as showed in Figure 5-13, a desired speed distribution for the vehicle inputs in VISSIM has been established. For each speed limit, a corresponding desired speed distribution has been set up.

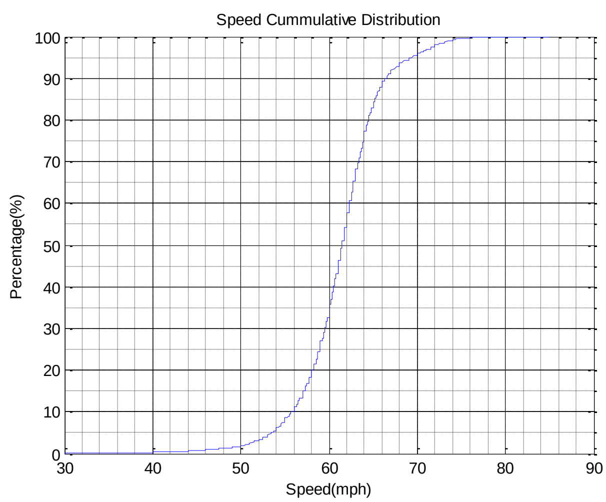

Figure 5-12 Cumulative speed distribution for real-field data

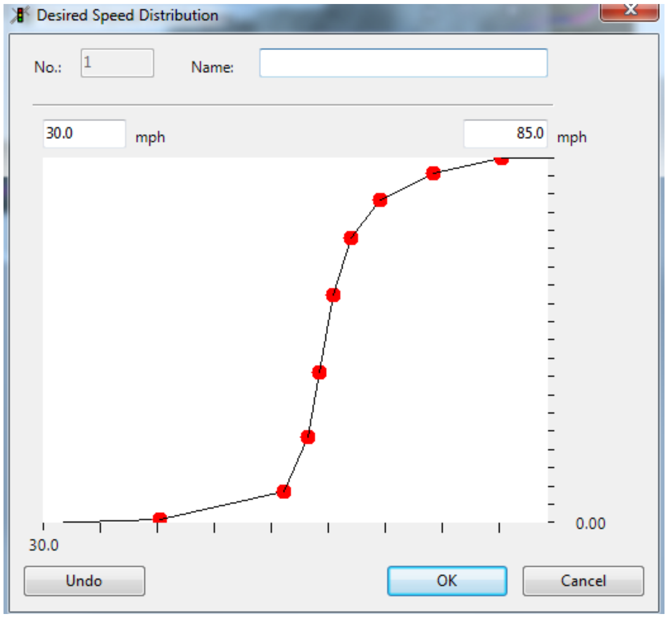

Figure 5-13 Desired speed distribution used in VISSIM

After setting up the required elements which represent real-field traffic composition, speed distribution, and traffic volumes, network calibration have been conducted based on 5-minute traffic volumes. In the previous simulation studies, network calibration and validation have been claimed as the most important and tedious procedure while conducting micro-simulation based transportation studies. Yadlapati and Park (2003) worked with un-calibrated network; other well calibrated studies (Chu *et al.*, 2003; Dhindsa, 2005; Nezamuddin *et al.*, 2011) utilized volumes at 5-minute interval and Geoffrey E. Heavers (GEH) statistic to compare the observed loop detector volumes with those captured in the simulation network. The GEH statistic, utilized by British engineers (UK Highway Agency, 1996) can be calculated as follows:

$$GEH = \sqrt{\frac{(M_{obs}(n) - M_{sim}(n))^2}{(M_{obs}(n) + M_{sim}(n))/2}}$$

where $M_{obs}(n)$ is the observed volume of in-field detectors and $M_{sim}(n)$ is the simulated volumes obtained from the simulation network. According to the Federal Highway Administration (FHWA), if more than 85% of the measurement locations' GEH values are less than 5, then the simulated flow would accurately reflect the real-field traffic flow (FHWA, 2004). Through tuning driver behavior parameters (vehicle following parameters and lane change parameters) in VISSIM, the microscopic simulation model was adjusted to replicate the real-field traffic conditions. Table 5-1 shows an example of GEH values of four detectors located in eastbound of the freeway section. From the table, it can be seen that during the three hours simulation period, more than 95% of the GEH values are less than 5. Results of the table indicate that the simulation model was satisfactorily calibrated for volumes. Moreover, multiple simulation runs (10 runs with distinct random seed) have been conducted to further confirm the calibration results.

Table 5-1 Sample profile of GEH values for calibration

5-min. Time Interval	GEH (MM 205.7)	GEH (MM 208)	GEH (MM 209.79)	GEH (MM 210.8)
1	0.289446	0.851179	0.390816	0.073727
2	1.494092	0.435561	3.369065	0.993793
3	0.807075	0.210255	1.191404	0.145256
4	0.194923	0.060271	1.046885	2.575122
5	3.34772	0.397572	1.791197	1.336425
6	0.934001	2.122882	0.446961	2.391514
7	0.161787	1.208237	2.428094	1.436474
8	1.411905	1.566342	2.406715	1.360564
9	1.894135	0.801483	0.991493	1.231458
10	1.212323	1.711463	0.686911	0.431376
11	1.867717	1.180044	0.644062	0.268539
12	7.093041	6.592078	4.259106	6.253796
13	1.174817	0.398333	1.52253	1.665445
14	0.563639	0.269083	1.504795	0.637025
15	0.419262	1.722133	3.181299	2.382845
16	0.623052	1.13418	2.633488	0.504936
17	1.956872	0.009494	1.195541	1.801561
18	0.465769	0.498424	1.755725	0.389051
19	3.658419	0.648444	2.209592	0.505924
20	0.873258	0.034496	1.084979	0.115986
21	1.912428	0.840565	1.494093	0.386429
22	3.706261	2.891816	0.576861	0.099413
23	1.614976	0.28284	0.789087	1.524778
24	0.659244	0.048737	2.521379	0.4509
25	1.603719	0.599733	0.293467	1.168762

5-min. Time Interval	GEH (MM 205.7)	GEH (MM 208)	GEH (MM 209.79)	GEH (MM 210.8)
26	1.690863	0.292344	1.784431	0.42083
27	1.584236	0.654711	2.44313	0.427005
28	0.669534	1.065936	1.778714	0.987399
29	0.849396	1.403002	3.318776	1.505781
30	0.627517	1.059785	0.545709	2.355196
31	0.237598	0.133611	1.815236	0.086441
32	1.765785	1.714366	2.695229	1.867678
33	2.800422	0.515276	2.050538	1.491913
34	0.466452	1.098246	0.974176	1.632277
35	2.135701	0.272835	2.336782	1.301225
36	0.373688	0.414151	0.46582	0.314462

5.3.3 Network validation

As the results showed in the table, the simulation model has been satisfactorily calibrated for volumes at 5-min interval. For the sake of validating the simulation network, average speeds from the in-field detectors have been utilized. Mean, minimum, and maximum values of average speed values at 5-min interval were calculated; simulation speeds data were extracted from the VISSIM simulation outputs. Speed profiles (Figure 5-14 - Figure 5-17) were utilized to compare the simulated speeds to the actual speeds (mean values, minimum, and maximum bounds) from in-field detectors.

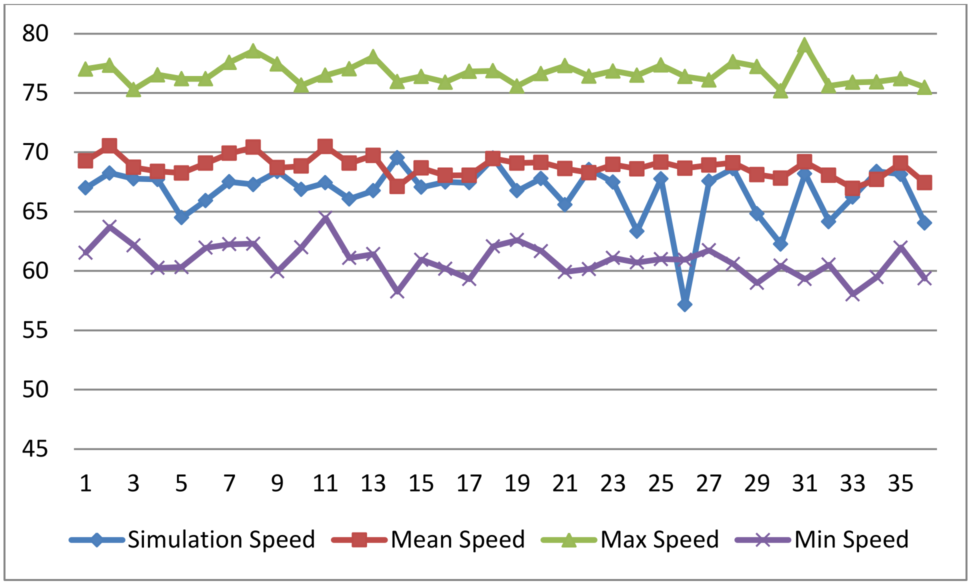

Figure 5-14 Speed comparisons for MM 205.7

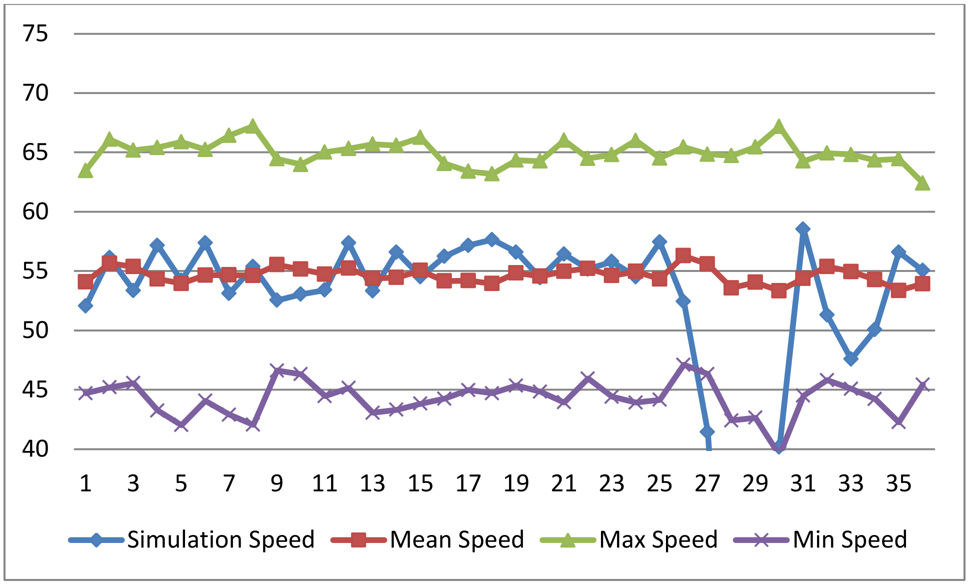

Figure 5-15 Speed comparisons for MM 208

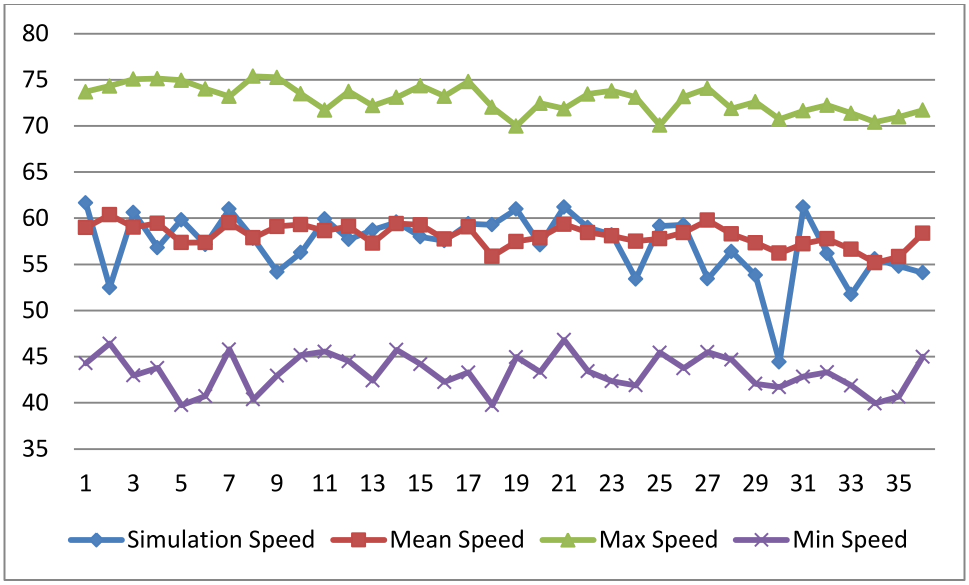

Figure 5-16 Speed comparisons for MM 209.79

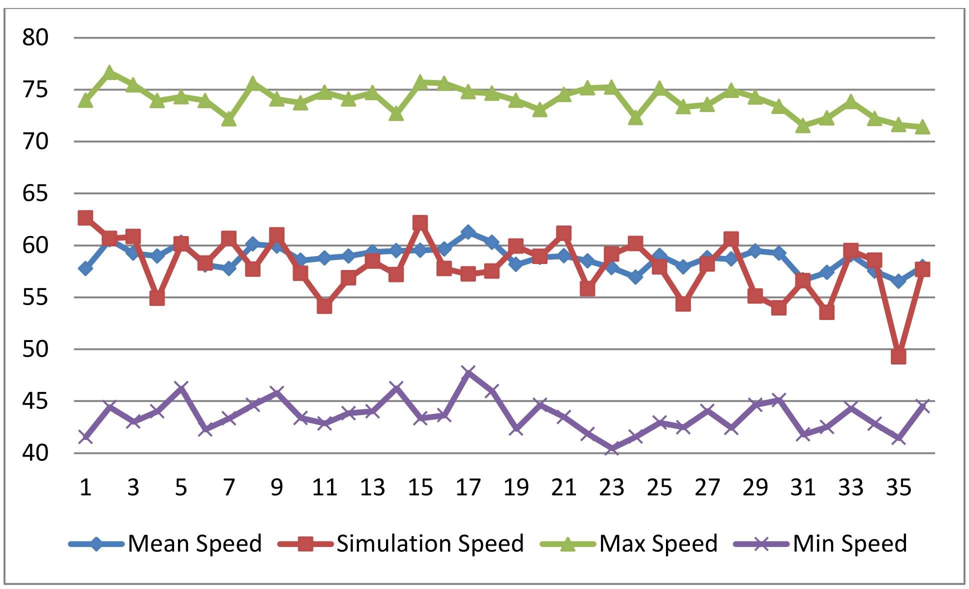

Figure 5-17 Speed comparisons for MM 210.8

Furthermore, besides visible inspections of the simulation speeds and actual speeds, quantified analyses have also been conducted. According to Nezamuddin *et al.* (2011), simulated speeds should be within the errors of 5 mph compared to in-field speeds for 85% of the checkpoints. Table 5-2 displays an example for errors of speeds at different locations between the simulation and in-field data. Results showed in the table indicate that speed values were satisfactorily validated, errors at different locations are within acceptable ranges.

Table 5-2 Speed errors for validation

5-min. Time Interval	Error (MM 205.7)	Error (MM 208)	Error (MM 209.79)	Error (MM 210.8)
1	-2.2887	-2.0352	2.680902	4.884292
2	-2.27542	0.495603	-7.87891	0.123938
3	-0.96586	-2.02231	1.608954	1.601047
4	-0.67426	2.833503	-2.63415	-4.08788
5	-3.76804	0.253934	2.48241	-0.14515
6	-3.14419	2.713851	-0.2288	0.180796
7	-2.40084	-1.55166	1.498725	2.895289
8	-3.13938	0.734162	-0.07552	-2.43125
9	-0.31538	-3.00233	-4.92703	1.057215
10	-1.9562	-2.10153	-3.035	-1.26052
11	-3.04568	-1.34231	1.277334	-4.66184
12	-3.00096	2.122558	-1.42068	-2.1162
13	-2.97746	-1.0648	1.422235	-0.89684
14	2.417477	2.138843	0.149071	-2.28617
15	-1.62298	-0.52204	-1.29082	2.650658
16	-0.54567	2.079003	-0.16816	-1.85458
17	-0.6382	2.970638	0.348936	-4.04162
18	0.02887	3.702477	3.41358	-2.77299
19	-2.33742	1.744821	3.528594	1.751988
20	-1.36051	-0.12016	-0.76619	0.081624
21	-3.0306	1.440619	1.879428	2.156846
22	0.260538	-0.06003	0.543185	-2.69916
23	-1.49254	1.196861	0.119913	1.306229
24	-5.25136	-0.441	-4.08266	3.213682
25	-1.41108	3.11632	1.385814	-1.10314
26	-11.5004	-3.83799	0.834506	-3.56404
27	-1.34852	-14.1377	-6.35039	-0.61934
28	-0.47245	-2.2222	-1.87966	1.915438
29	-3.30168	-5.9119	-3.51907	-4.34502
30	-5.56391	-3.1666	-11.794	-5.26949
31	-0.98329	4.158727	3.958825	-0.08418
32	-3.90016	-4.08633	-1.58905	-3.86836

5-min. Time Interval	Error (MM 205.7)	Error (MM 208)	Error (MM 209.79)	Error (MM 210.8)
33	-0.7379	-7.36561	-4.88014	0.400232
34	0.673082	-4.22508	0.421267	1.048314
35	-0.94516	3.24247	-1.0115	-7.27671
36	-3.38537	1.113973	-4.25223	-0.30028

CHAPTER 6: SIMULATION SETTINGS AND RESULTS ANALYSES

6.1 VISSIM setting

For the purpose of testing the feasibility of proposed VSL control algorithm in improving traffic safety, bottleneck area on the eastbound of I-70 has been chosen as the VSL control section. The studied area starts from Mile Marker (MM) 211.75 and ends at MM 214 where four VSL signs and six detectors are virtually implemented in VISSIM. VSL control areas are defined as segments between two neighboring VSL signs with detectors implemented in the middle of each segment.

The freeway section changes from three lane sections to two lane sections where the merge happens at MM 213.1. Figure 6-1 presents the locations of the VSL signs (triangles), detectors (circles), and the merge point (square). Relative to the bottleneck location, four VSL signs are named after VSL U3, VSL U2, VSL U1, and VSL D1 respectively; U and D represent the upstream and downstream whereas the upstream and downstream are defined with respect to the bottleneck area. The six detectors are called as detector U4, U3, U2, U1, D1 and D2 accordingly; no on- and off-ramps exist for the studied area.

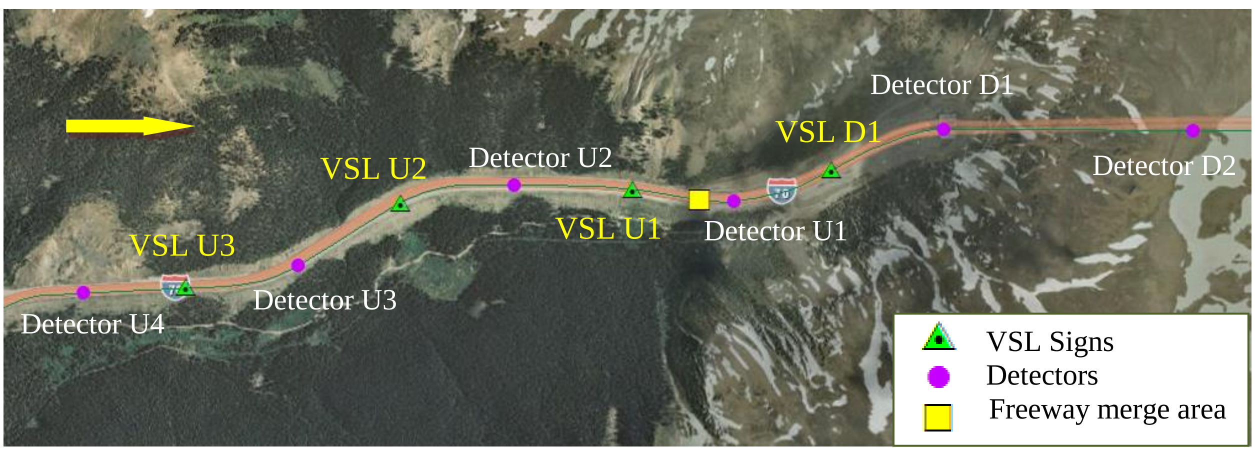

Figure 6-1: Locations of the VSL signs, detectors and merge point (1:15000)

'Desired speed decision points' are utilized to alter the speed limits in VISSIM; various speed limits come into service by assigning different 'desired speed distributions'. In order to test the proposed VSL algorithm in VISSIM, speed distributions under various speed limits need to be defined. The original speed limits for the studied freeway section are 60 mph and 50 mph; in-field traffic data under these two speed limits are available. In order to identify the speed distributions under other speed limits, a heuristic approach to interpolate speed distributions under the speed limits of 55 mph, 45 mph, and 40 mph is adopted. First, speed distributions under the speed limits of 60 mph and 50 mph are obtained from the real-time traffic data archived by the RTMS radars. Therefore, PROC SEVERITY procedure in SAS (SAS Institute, 2004) is employed to identify distributions of average speed: Normal, Gamma, Exponential, Lognormal, and Weibull distribution are the candidate distributions. Maximum likelihood method is used to estimate parameters of distributions in the procedure. The likelihood-based statistics Akaike's Information Criterion (AIC) are supplied to indicate the fittings of the estimated distributions and to identify the most appropriate distributions. The smaller the AIC value is, the better the distribution fits the data. Table 6-1 displays the distribution fitting results for the average speed under the speed limit of 60 mph.

Table 6-1: Speed distributions fitting results for the speed limit 60 mph

Distribution	Converged	AIC	Selected
Normal	Yes	1544	No
Exponential	Yes	2021	No
Gamma	Yes	1582	No
Lognormal	Yes	1613	No
Weibull	Yes	1529	Yes

Previous study (Hellinga and Mandelzys, 2011) employed normal distributions to represent the speed distributions. However, results from Table 6-1 demonstrate that Weibull distribution best fits the speed; normal distribution provides the second best fit. The best fitted Weibull distribution is selected to represent speed distributions for the studied freeway section. Parameters describing the Weibull distributions are also provided by SAS. By interpolating the distribution parameters, speed distributions under the speed limits of 55 mph, 45 mph, and 40 mph are obtained. Table 6-2 provides the parameters of Weibull distributions under different speed limits; Figure 6-2 to Figure 6-6 presents the probability density function (PDF) plots for the speed limits of 60, 55, 50, 45, and 40 respectively. It is worth noting that in Table 6-2 and Figures, speed distributions of speed limit 60 and 50 are extracted from in-field data while distributions under other speed limits are achieved through interpolation.

Table 6-2: Weibull distribution parameters for different speed limits

Distribution	Speed Limit (mph)	θ*	τ
Weibull	60	61.66	6.21
Weibull	55	54.84	6.3
Weibull	50	51.22	6.59
Weibull	45	46.77	6.6
Weibull	40	41.35	6.8

*PDF for the Weibull distribution $f(x) = \frac{1}{x}\tau(\frac{1}{\theta})^{\tau}e^{-(\frac{1}{\theta})^{\tau}}$

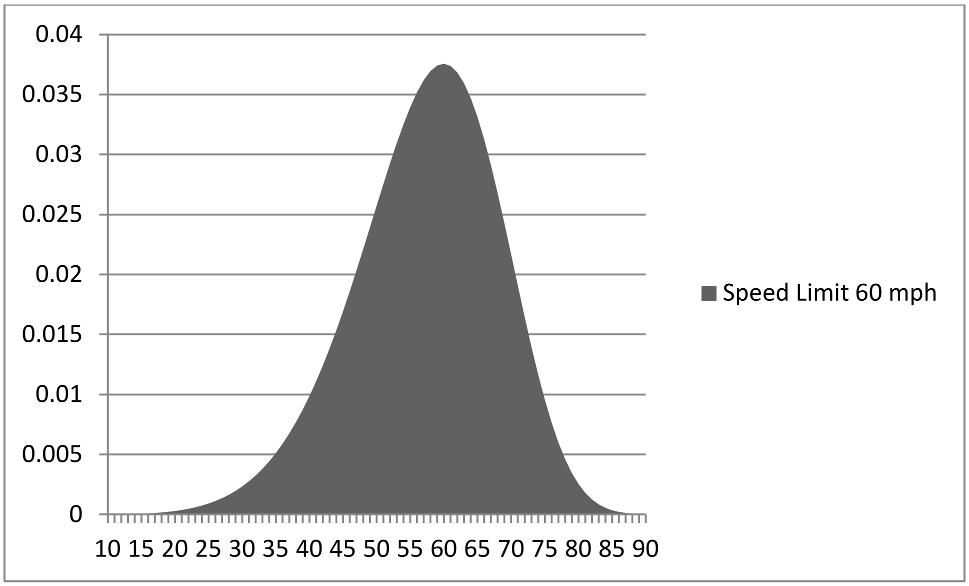

Figure 6-2 PDF plot for speed limit of 60 mph

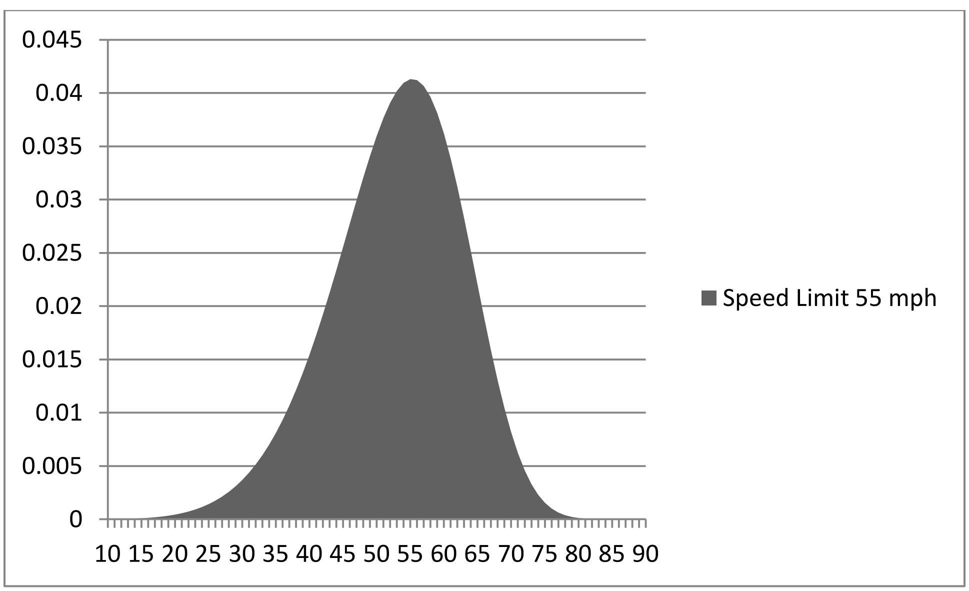

Figure 6-3 PDF plot for speed limit of 55 mph

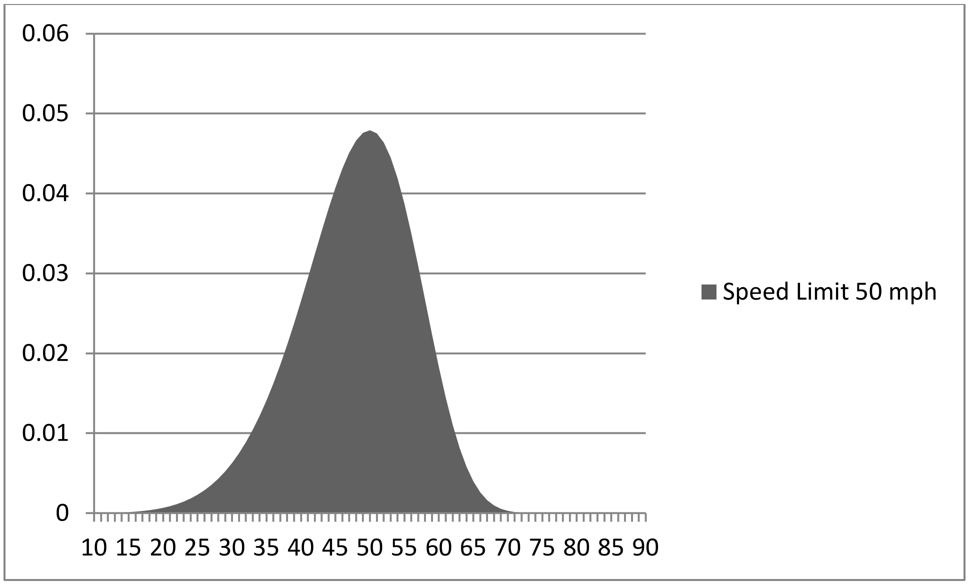

Figure 6-4 PDF plot for speed limit of 50 mph

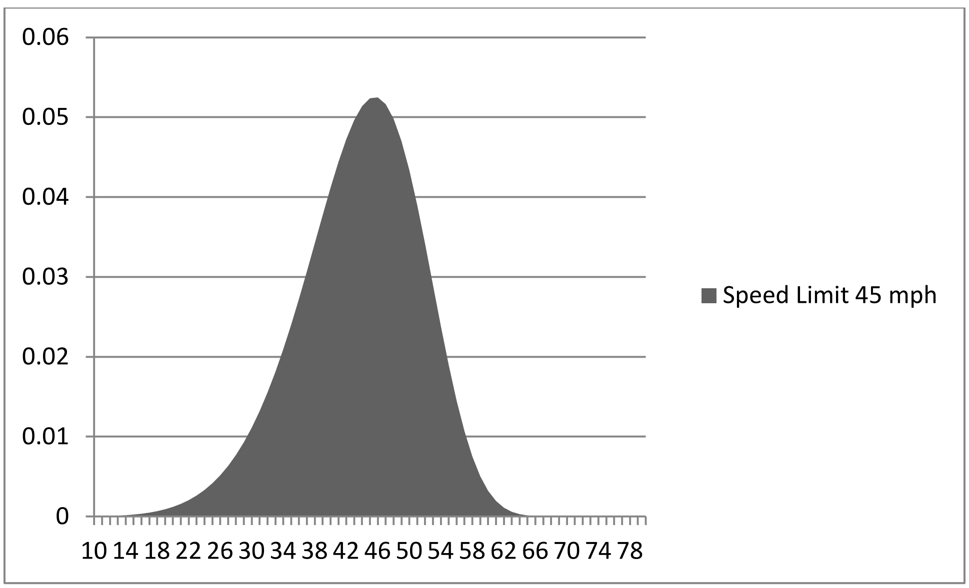

Figure 6-5 PDF plot for speed limit of 45 mph

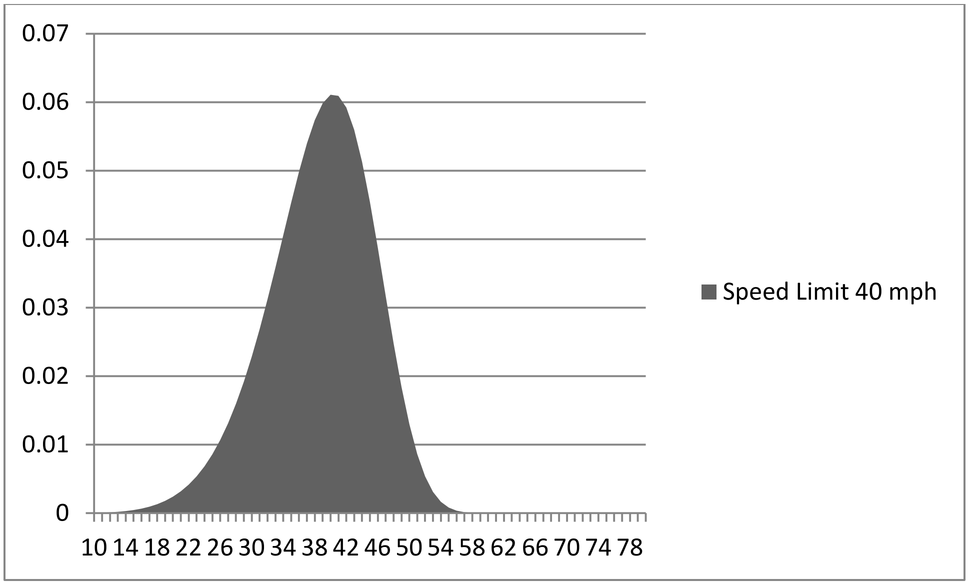

Figure 6-6 PDF plot for speed limit of 40 mph

The abovementioned results of interpolating speed distributions are achieved under the assumption that drivers would keep the existing compliance level for variable speed limits. However, as indicated by Hellinga and Mandelzys (2011), the traffic safety improvements created by VSL are especially sensitive to the drivers' compliance levels. By assuming that the speed distributions would remain the same type, three compliance levels are created by altering parameters of the Weibull distributions, which are shown in Table 6-3. The proposed VSL system would be tested under these three compliance levels. We should note that the speed distributions described above correspond to the high compliance level in Table 6-3.

Table 6-3: Expected mean free-flow speed (mph) of different speed limit compliance levels

	Compliance Level		
Speed Limits	Low	Moderate	High
55	66	60	55
50	61	55	49
45	55	50	45
40	50	45	39

6.2 METANET Model

Parameters in the METANET model are calibrated based on synthetic data captured by the detectors in VISSIM. The link specified parameters are $v_{f,m} = 65\ mph,\ \rho_{cr,m} = 26$ veh/lane/

mile, $a_m = 1.41, \tau = 0.015\ hr,\ \kappa = 64$ veh/lane/mile and $\eta = 25\ mile^2/h$. Moreover, considering the VSL impacts on the fundamental impacts, the parameters A_m and E_m are also calibrated with the simulation data under different speed limits and various compliance levels. Finally, as displayed in Table 6-4, these two parameters are estimated based on the compliance levels.

Table 6-4: VSL related parameters

Compliance Level	A_m	E_m
Low	0.72	1.73
Moderate	0.71	1.69
High	0.65	1.85

6.3 Crash risk evaluation model

There are two datasets utilized to calibrate the crash risk evaluation model: (1) crash data from Oct 2010 to Oct 2011 provided by the Colorado Department of Transportation (CDOT) and (2) real-time traffic data detected by 30 RTMS radars. There are 265 crashes documented and matched with traffic data during the studied period; 1017 non-crash cases are selected and matched to the crash cases. The RTMS radars archived speed and volume information at 30-second intervals. Traffic status corresponding to each crash is prepared by extracting traffic data 5-10 minute prior to the crash occurrence time at the crash location. The 5-10 minute time period prior to the reported crash time is utilized to avoid confusing pre and post crash conditions. For example, if a crash happened at 15:25, at MM 211.3, the corresponding traffic status for this crash would be traffic conditions of time interval 15:15 and 15:20 recorded by RTMS radar at MM 211.8. To coordinate the crash risk evaluation model with the METANET traffic flow model, only the 5-minute average speed, average density, and total volume are considered as candidate explanatory variables in the crash risk evaluation model.

For each specific crash case, four non-crash cases are identified and matched. The non-crash cases are selected based on the following procedures: for example, a crash happened on Tuesday (May 24, 2011), then the four non-crash cases would be selected for the exact same time interval two weeks before and two weeks after the crash time (May 10, May 17, May 31, and Jun 7) at the exact location of crash occurrence. This data preparation approach utilized matched case-control design, which is frequently employed in the disaggregate crash occurrence studies;

confounding factors can be controlled through matching (Breslow and Day, 1980). This matched case-control structure would implicitly account for the influences of geometric characteristics, peak hour effects, driver population, etc, on crash risk evaluation.

The CR model is estimated in SAS with the PROC LOGISTIC procedure. Table 6-5 shows results of the logistic regression model. Among the three candidate explanatory variables, only the average speed variable is found to be significant. The negative sign for average speed indicates that crashes are more likely to occur within congested areas and periods. The result is consistent with the CR models previously estimated for the same roadway section (Ahmed *et al.*, 2012a) and also expressways in other jurisdiction (Ahmed *et al.*, 2012b). Moreover, ROC Index (0.74) demonstrates that the estimated model could satisfactorily classify crash and non-crash cases.

Table 6-5: Crash risk evaluation model

Parameter	Estimate	Standard Error	Wald Chi-Square	P-value
Intercept	1.98	0.31	39.5	<0.0001
Average speed	-0.067	0.006	116.2	<0.0001
AIC		1180.84		
ROC Index		0.74		

With the estimated CR model, the predicted crash risks for time period $(k+1)$ can be calculated with the following equation:

$$Crash\ Risk_{m,i}(k+1) = \frac{\exp(1.98 - 0.067 * v_{m,i}(k+1))}{1 + \exp(1.98 - 0.067 * v_{m,i}(k+1))}$$

6.4 Simulation Results

The proposed VSL algorithm is implemented through the VISSIM component object model interface with a module developed with C++ program. In order to establish a system that has low sensitivity to prediction errors, the VSL rates $b_m(k+1)$ are calculated at every 5-min time step. Ten runs, each with a different random seed value, are conducted for each compliance level scenario. Table 6-6 shows an example of final VSL control strategy with the high compliance level and random seed of 77.

Table 6-6: Example of VSL control strategies (high compliance, random seed 77)

Time Interval (5-min)	VSL U3	VSL U2	VSL U1	VSL D1
1-17	60	60	60	60
18	50	60	60	60
19	45	45	45	55
20	45	45	45	55
21	55	55	55	50
22	60	60	60	60
23	60	60	60	60
24	60	60	60	60
25	50	50	50	50
26	45	45	45	50
27	55	50	45	50
28	60	60	55	60
29-36	60	60	60	60

Safety effects of the proposed VSL system are quantified as crash risk improvements and speed homogeneity improvements. Figure 6-7 and 6-8 display the average crash risk improvements (negative means traffic safety improved) and the speed homogeneity improvements (negative means smaller speed standard deviations have been achieved) with three compliance levels, accordingly.

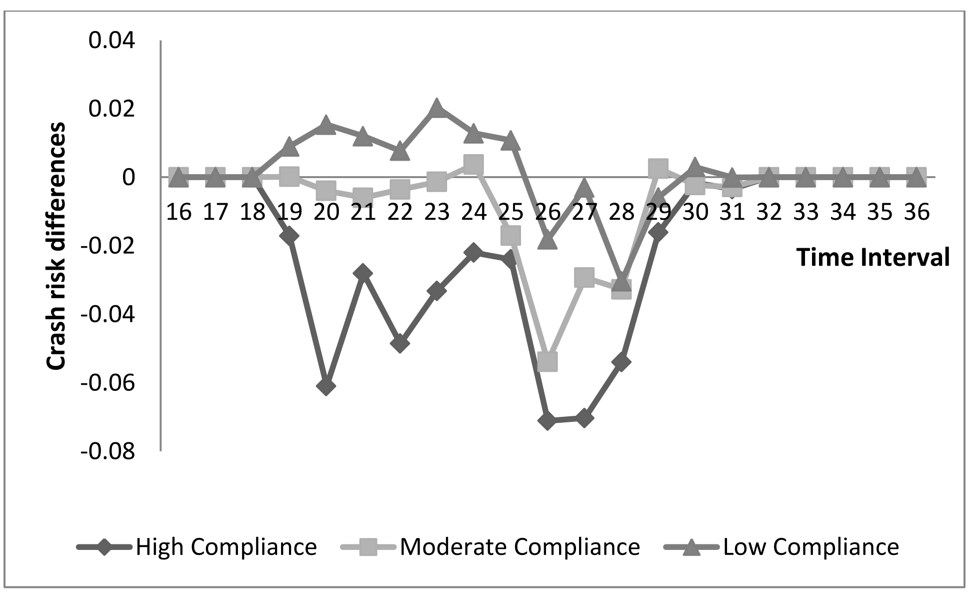

Figure 6-7: Average crash risk improvements for three compliance levels

Figure 6-7 shows the crash risk differences of the VSL cases compared to the non-VSL control cases. Negative values of crash risk improvements indicate enhanced traffic safety with VSL; while positive crash risk differences mean worse traffic safety situations. From Figure 6-7 it can be seen that with high and moderate compliance levels, crash risks have been decreased for almost all the VSL implemented periods; while for the low compliance level, mixed crash risk is achieved with the triggered VSL system (increase in particular during the time interval of 21-25). This phenomenon confirmed that effects of VSL on traffic safety vary across the compliance levels (Hellinga and Mandelzys, 2011).

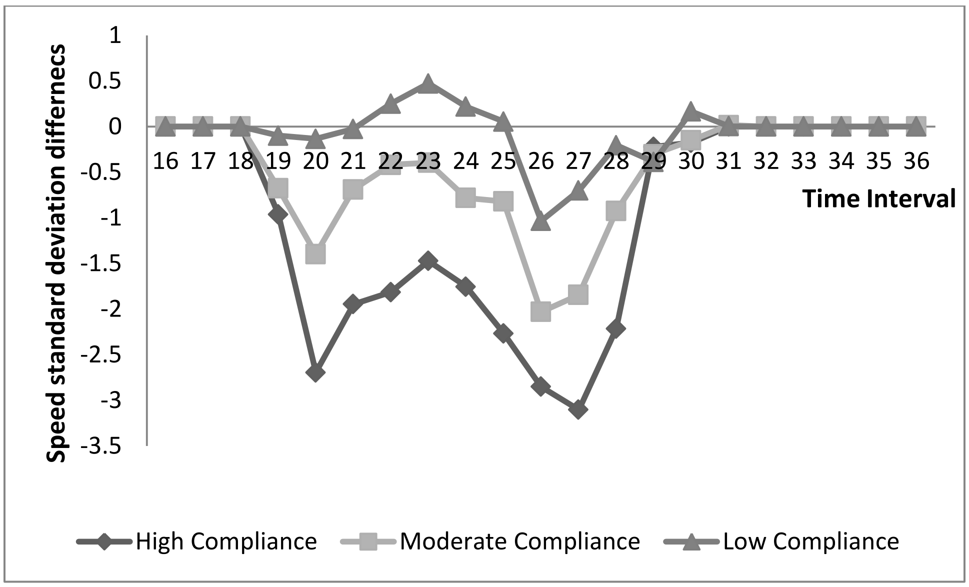

Figure 6-8: Average speed standard deviation improvements for three compliance levels

Furthermore, standard deviation of average speed is utilized as a measurement of homogeneity (Nes *et al.*, 2010), which is used as another evaluation measurement for traffic safety. Figure 6-8 displays the differences of speed standard deviations between the non-VSL base case and VSL control cases with various driver compliance levels. Negative values for speed standard deviation differences indicate smoother and homogeneous traffic flow while positive dots indicate turbulent traffic. Similar conclusions can be drawn by monitoring the speed homogeneity: improved speed homogeneities have been achieved by implementing the VSL system under the high and moderate driver compliance levels; if the majority of the drivers ignore the lowered speed limits, speed homogeneity would deteriorate. It is worth mentioning that speed harmonization is one of the main objectives of ATM.

However, as indicated by the existing speed patterns (speed distributions with speed limit 60 mph and 50 mph), drivers' compliance mains a relatively high level for the existing speed limits. Assuming the same compliance level would persist for VSL, significant traffic safety improvements can be achieved by the proposed VSL control algorithm. Nevertheless, previous figures showed only averaged crash risks and averaged speed standard deviations of the four VSL segments, crash risk migration issue cannot be detected. In order to make sure that VSL would not lower the crash risk at one location while increasing it at others, crash risk and speed standard deviation improvements with high compliance level of every detector are plotted in Figures 6-9 and 6-10, respectively; which is the same method utilized in a previous study (Abdel-Aty *et al.*, 2006b). In this study, a total of six detectors are incorporated to fully investigate the crash risk migration issue. From the figures, it can be observed that crash risk migration issue has been effectively prevented. Additionally, the percentages of improved crash risk, speed homogeneity, and travel time improvement relative to the non-VSL conditions for each location are listed in Table 6-7. Values in the table further confirm that VSL control has no negative effects on crash risk and speed homogeneity improvements for the entire freeway section. In addition, except for the segment of U4, average travel times for the VSL control section have been decreased.

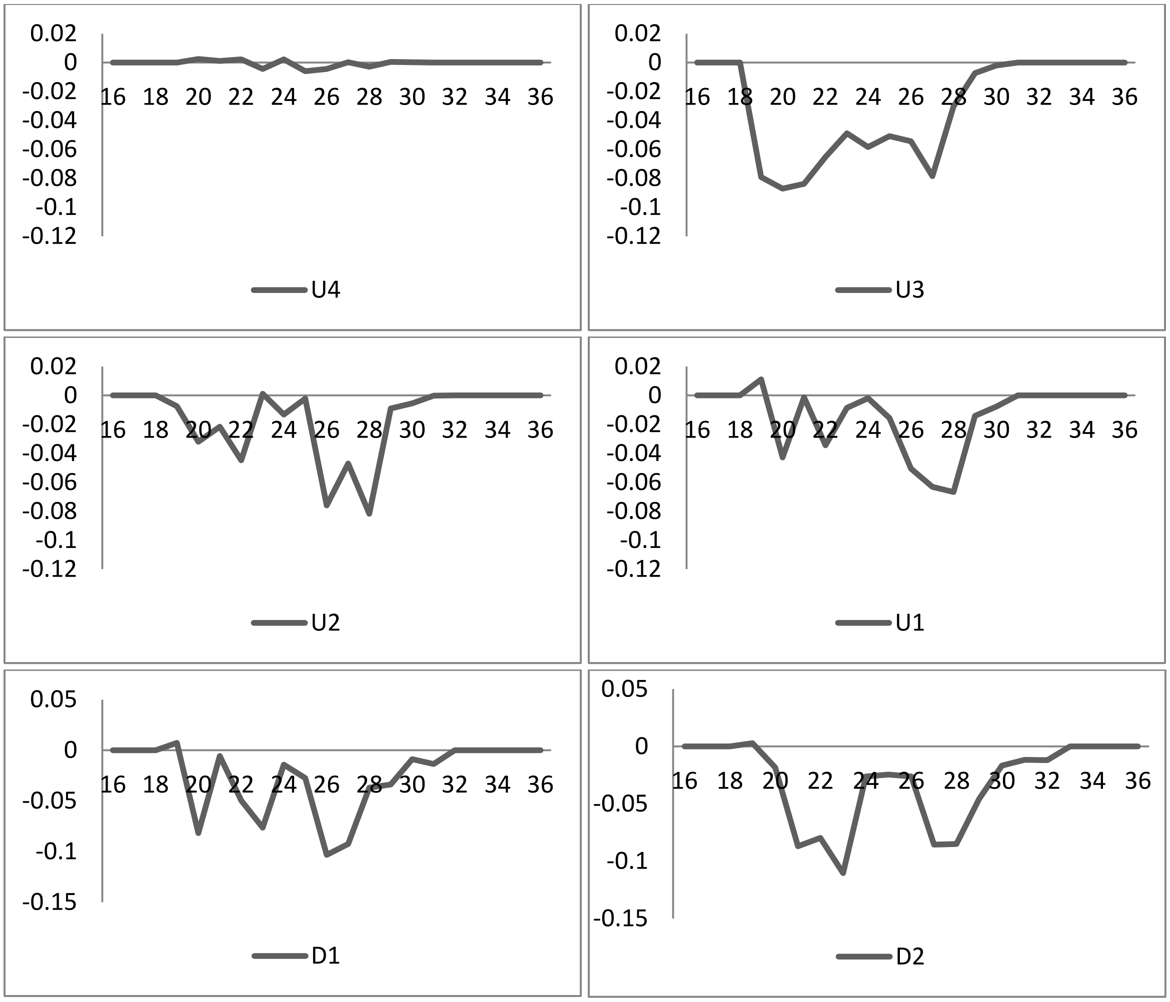

Figure 6-9: Crash risk improvements for different locations

Table 6-7: Percentages of crash risk and speed homogeneity improvements for each location

Detector Location	Crash risk improvement	Speed homogeneity improvement	Travel time improvement
Location U4	0.19%	2.9%	-3.6%
Location U3	13.1%	17.9%	5.7%
Location U2	7%	11.4%	2.3%
Location U1	6.2%	12.5%	2.6%
Location D1	11.8%	11.4%	4.7%
Location D2	13.1%	8.4%	5.2%

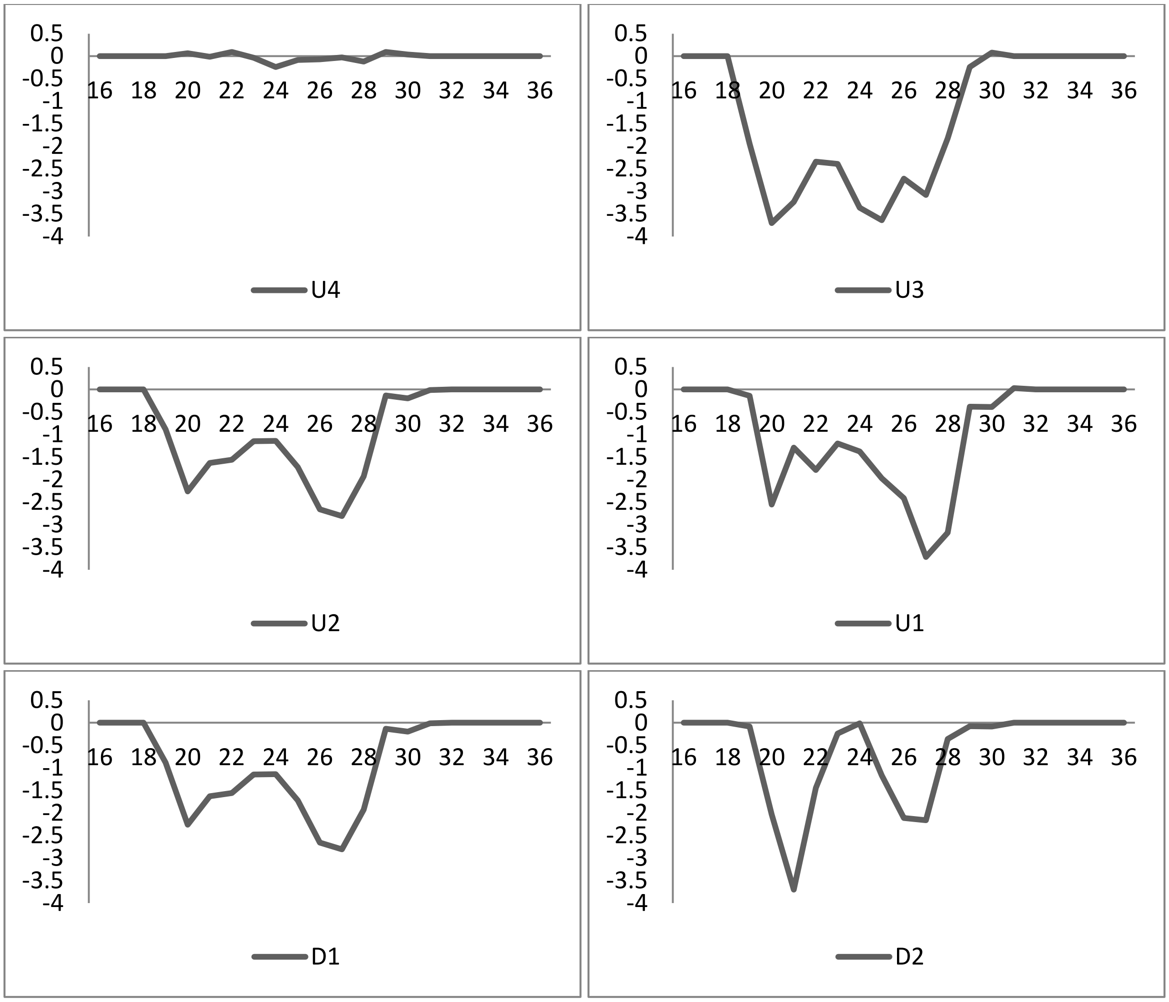

Figure 6-10: Speed standard deviations' improvements for different locations

CHAPTER 7: CONCLUSIONS AND RECOMMENDATIONS

7.1 Conclusions

This study focuses on proposing an innovative approach to identify an optimal VSL control strategy with the purpose of pro-actively improving traffic safety on freeways. Firstly, the state-of-practice VSL systems implemented in Europe and US have been reviewed for the aspects of system objectives, control algorithms, data collection and speed limits displaying devices, evaluation methods, and evaluation results. Then the state-of-art research studies have also been investigated for their advanced control algorithms. It can be concluded that previous studies aimed at improving traffic safety with VSL systems (Abdel-Aty *et al.*, 2006a; Lee *et al.*, 2006b) identified the best control strategies through an experimental design approach; through testing all possible control scenarios, the best control strategies that can reduce crash risk were identified. This approach is time consuming and the results are not transferrable among different facilities. Other VSL studies (Hegyi *et al.*, 2005; Carlson *et al.*, 2010a) intended to improve traffic flow (minimizing total travel time) obtained the best control strategies by solving optimization problems. However, traffic safety was not considered during the optimization procedures. In this study, a VSL control algorithm which bridges the gap by combining the traffic flow model with a real-time crash risk evaluation model has been introduced. The VSL control algorithm is designed to minimize total crash risk along the studied roadway section while also controlling for the average travel time.

The VSL control algorithm utilized in this study contains two major parts: an extension of METANET traffic flow model and a crash risk evaluation model. The METANET model is introduced to analyze VSL effects on traffic flows while the crash risk evaluation model is developed to quantify the traffic safety risk. Optimal control strategies are obtained through an optimization framework for the purpose of minimizing crash risk along the VSL control area. In addition, constraints of average travel time increase are defined to ensure the reliability of travel time and insure preventing unacceptable travel time increases.

To test the proposed VSL control algorithm, a bottleneck area on eastbound I-70 is carefully coded in the micro-simulation software VISSIM. Four VSL signs are implemented as three signs upstream and one downstream of the bottleneck. Detectors are installed in the middle of two neighboring VSL signs. Traffic flow scenarios during morning peak hours in August, 2011 (9-11

AM) are calibrated and validated with real field data. The optimal VSL control strategies are implemented through COM in VISSIM. A total of three different driver compliance levels are investigated; each compliance level is simulated with ten different random seed numbers. Results of the VSL are quantified as average crash risk improvements and average speed homogeneity improvements across the ten simulation runs. From the simulation results, it can be concluded that VSL would effectively improve traffic safety under high and moderate compliance levels; while with low compliance level, the results are mixed. In addition, possible crash migration phenomena have been fully investigated by plotting crash risks versus six detector locations; no crash risk migration issue was detected.

7.2 Recommendations

Abovementioned results demonstrate that the proposed VSL control algorithm could effectively improve traffic safety without increasing average travel time; there are still improving spaces for the existing algorithm. In this study, candidate explanatory variables used in the crash risk evaluation model are average speed, density, and volume as provided by the METANET model. However, as for a mountainous freeway, crash occurrence is also substantially influenced by weather conditions (Yu and Abdel-Aty, 2013;Yu *et al.*, 2013). Visibility and precipitation conditions also play critical roles in crash occurrence. Due to the weather conditions cannot be reflected in the simulation software, future implementations of the proposed VSL system should consider including weather related variables into the crash risk evaluation model. Besides, the METANET model should also be improved by considering VSL effects on traffic flow during various weather conditions.

In addition, this study utilized one simple crash risk evaluation model for the total crashes to evaluate traffic safety. However, as stated in previous studies (Pande and Abdel-Aty, 2006a; Yu *et al.*, 2013), it is important to analyze the crash by types, particularly when it comes to real-time crash risk assessment. Future studies may develop multiple crash risk evaluation models for single-vehicle and multi-vehicle crashes; where the optimization problem would be extended to a multi-objective optimization problem with balanced traffic safety improvements for different crash types. Besides, as concluded by Yu and Abdel-Aty (2013) that support vector machine technique would perform better than the classic logistic regression models as crash risk evaluation measure technique. Instead of utilizing logistic regression models, more advanced crash risk evaluation models can be employed in future studies.

Moreover, as this study mainly focused on the control algorithm of the VSL system, other perspectives of the system also need further investigation, such as the compliance issue. As indicated by the results in this study that effects of the VSL system on traffic safety vary by the compliance levels. Studies regarding to how to educate the drivers should be conducted. Furthermore, different displaying devices may result in various compliance levels; what kind of ITS devices should be utilized for VSL to convey the changeable speed limits message to the drivers should also be investigated.

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APPENDIX

Results of the ten paired simulation runs (with and without VSL control) in VISSIM with high compliance level are shown in the appendix; calculated crash risk for six locations are listed.

Without VSL (Random Seed 5)

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
1	0.109449	0.164701	0.159934	0.160247	0.176781	0.153181
2	0.105036	0.132744	0.139724	0.147061	0.148526	0.150936
3	0.108753	0.168089	0.153777	0.151648	0.166536	0.165888
4	0.119061	0.139883	0.144761	0.148377	0.160980	0.179658
5	0.111921	0.171122	0.172879	0.142089	0.153896	0.159239
6	0.144524	0.171135	0.155962	0.160071	0.187528	0.159058
7	0.109442	0.158094	0.191987	0.186740	0.214014	0.228906
8	0.130077	0.169558	0.150978	0.153068	0.216650	0.237174
9	0.210720	0.231130	0.169048	0.165044	0.187975	0.215993
10	0.130842	0.196944	0.275210	0.249257	0.218645	0.181412
11	0.161794	0.170617	0.191644	0.157088	0.362350	0.271370
12	0.117863	0.211145	0.220344	0.184165	0.188026	0.185702
13	0.122791	0.178954	0.166322	0.211664	0.308881	0.213998
14	0.164924	0.217523	0.186504	0.167356	0.221089	0.250439
15	0.169915	0.231280	0.178612	0.196628	0.243408	0.191938
16	0.196891	0.202144	0.220821	0.251091	0.230783	0.259147
17	0.120911	0.202860	0.229076	0.181345	0.251421	0.263841
18	0.147351	0.205486	0.218772	0.211323	0.238748	0.208135
19	0.139526	0.299262	0.220948	0.228851	0.282331	0.251991
20	0.325341	0.372067	0.292300	0.325844	0.316455	0.258609
21	0.153042	0.318743	0.402792	0.386254	0.260071	0.232623
22	0.155313	0.325176	0.285013	0.257769	0.451541	0.430775
23	0.587224	0.516095	0.228190	0.262285	0.273668	0.288893
24	0.215999	0.509546	0.651493	0.507595	0.294208	0.279620
25	0.634762	0.543875	0.300518	0.393111	0.563234	0.498197
26	0.404382	0.578242	0.631168	0.458471	0.469587	0.394832
27	0.314389	0.455507	0.485372	0.509417	0.624763	0.618986
28	0.583313	0.565755	0.426893	0.390389	0.410859	0.483968

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
29	0.253789	0.348534	0.530301	0.536415	0.305672	0.273402
30	0.189304	0.248843	0.199094	0.242240	0.472272	0.367120
31	0.186515	0.258061	0.243294	0.232980	0.309374	0.310929
32	0.146630	0.209115	0.193850	0.283605	0.292059	0.324964
33	0.199382	0.161943	0.172569	0.187980	0.270527	0.217907
34	0.356669	0.355582	0.297480	0.261761	0.266088	0.227030
35	0.148558	0.198922	0.243034	0.280126	0.353892	0.251990
36	0.184646	0.232767	0.212361	0.227014	0.225622	0.257945

With VSL (Random Seed 5)

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
1	0.109449	0.164701	0.159934	0.160247	0.176781	0.153181
2	0.105036	0.132744	0.139724	0.147061	0.148526	0.150936
3	0.108753	0.168089	0.153777	0.151648	0.166536	0.165888
4	0.119061	0.139883	0.144761	0.148377	0.16098	0.179658
5	0.111921	0.171122	0.172879	0.142089	0.153896	0.159239
6	0.144524	0.171135	0.155962	0.160071	0.187528	0.159058
7	0.109442	0.158094	0.191987	0.18674	0.214014	0.228906
8	0.130077	0.169558	0.150978	0.153068	0.21665	0.237174
9	0.21072	0.23113	0.169048	0.165044	0.187975	0.215993
10	0.130842	0.196944	0.27521	0.249257	0.218645	0.181412
11	0.161794	0.170617	0.191644	0.157088	0.36235	0.27137
12	0.117863	0.211145	0.220344	0.184165	0.188026	0.185702
13	0.122791	0.178954	0.166322	0.211664	0.308881	0.213998
14	0.164924	0.217523	0.186504	0.167356	0.221089	0.250439
15	0.169915	0.23128	0.178612	0.196628	0.243408	0.191938
16	0.196891	0.202144	0.220821	0.251091	0.230783	0.259147
17	0.120911	0.20286	0.229076	0.181345	0.251421	0.263841
18	0.147351	0.205486	0.218772	0.211323	0.238748	0.208135
19	0.139526	0.219771	0.233082	0.206408	0.274111	0.271829
20	0.322252	0.251908	0.181481	0.191317	0.269854	0.267958
21	0.153042	0.281305	0.321513	0.267312	0.158233	0.16819

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
22	0.155238	0.281308	0.23854	0.214807	0.483017	0.356768
23	0.587224	0.515893	0.177742	0.255827	0.392789	0.220461
24	0.215999	0.509546	0.651493	0.507595	0.235614	0.254654
25	0.634762	0.543875	0.300518	0.393111	0.563234	0.498197
26	0.406805	0.542061	0.593108	0.462125	0.489728	0.42599
27	0.314694	0.462455	0.307812	0.414874	0.656913	0.559354
28	0.579382	0.561782	0.408858	0.309661	0.431113	0.458173
29	0.25469	0.309465	0.557905	0.520664	0.342757	0.294904
30	0.189304	0.248843	0.204379	0.263196	0.473917	0.380303
31	0.186515	0.258061	0.243294	0.23298	0.320177	0.298031
32	0.14663	0.209115	0.19385	0.283605	0.292059	0.324964
33	0.199382	0.161943	0.172569	0.18798	0.270527	0.217907
34	0.356669	0.355582	0.29748	0.261761	0.266088	0.22703
35	0.148558	0.198922	0.243034	0.280126	0.353892	0.25199
36	0.184646	0.232767	0.212361	0.227014	0.225622	0.257945

Without VSL (Random Seed 7)

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
1	0.10713	0.153685	0.150065	0.145476	0.152223	0.161844
2	0.104966	0.12295	0.120205	0.119613	0.137712	0.141676
3	0.1146	0.169679	0.165201	0.172567	0.168546	0.153834
4	0.105752	0.152336	0.166529	0.147376	0.180408	0.200257
5	0.202807	0.217193	0.129048	0.140007	0.178347	0.171485
6	0.187859	0.18145	0.261282	0.244996	0.16667	0.153475
7	0.112124	0.235219	0.259951	0.185718	0.488001	0.645514
8	0.172124	0.191807	0.157252	0.192177	0.288582	0.24388
9	0.121325	0.165658	0.174615	0.176503	0.184695	0.213909
10	0.192242	0.194343	0.153947	0.152301	0.267148	0.180406
11	0.123634	0.201778	0.264462	0.202591	0.175305	0.166895
12	0.129239	0.164877	0.161518	0.210169	0.388562	0.239188
13	0.165526	0.225382	0.205194	0.167114	0.229397	0.21988
14	0.157517	0.200934	0.184091	0.175484	0.245564	0.233888

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
15	0.124883	0.169943	0.208905	0.240994	0.365432	0.260744
16	0.141917	0.243729	0.170936	0.160289	0.308915	0.295827
17	0.108669	0.157099	0.184539	0.195782	0.258701	0.217463
18	0.172547	0.194879	0.140661	0.136071	0.219904	0.276702
19	0.167056	0.261928	0.188423	0.186766	0.162487	0.161585
20	0.228039	0.364195	0.318784	0.329868	0.251641	0.202539
21	0.381234	0.318539	0.315333	0.274901	0.397225	0.334679
22	0.631347	0.62792	0.457231	0.297197	0.383415	0.359765
23	0.587894	0.64581	0.63662	0.478983	0.357067	0.329157
24	0.237497	0.568369	0.645163	0.53154	0.496001	0.468389
25	0.296231	0.343234	0.281787	0.432849	0.654861	0.458466
26	0.386535	0.292156	0.35002	0.417407	0.452945	0.511195
27	0.429437	0.572372	0.518664	0.386284	0.539721	0.579948
28	0.15128	0.246715	0.458561	0.472419	0.414928	0.423764
29	0.140684	0.247886	0.249122	0.238042	0.291558	0.415097
30	0.349623	0.329912	0.219903	0.255328	0.290877	0.223427
31	0.213785	0.402799	0.40365	0.250496	0.354423	0.269284
32	0.398905	0.175069	0.282241	0.368313	0.499888	0.263402
33	0.210275	0.436551	0.430345	0.184794	0.255177	0.300688
34	0.55388	0.498903	0.30813	0.366041	0.359949	0.243389
35	0.127237	0.470472	0.605845	0.479511	0.346718	0.241392
36	0.153614	0.173807	0.192361	0.360788	0.380822	0.370894

With VSL (Random Seed 7)

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
1	0.10713	0.153685	0.150065	0.145476	0.152223	0.161844
2	0.104966	0.12295	0.120205	0.119613	0.137712	0.141676
3	0.1146	0.169679	0.165201	0.172567	0.168546	0.153834
4	0.105752	0.152336	0.166529	0.147376	0.180408	0.200257
5	0.202807	0.217193	0.129048	0.140007	0.178347	0.171485
6	0.187859	0.18145	0.261282	0.244996	0.16667	0.153475
7	0.112124	0.235219	0.259951	0.185718	0.488001	0.645514

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
8	0.172124	0.191807	0.157252	0.192177	0.288582	0.24388
9	0.121325	0.165658	0.174615	0.176503	0.184695	0.213909
10	0.192242	0.194343	0.153947	0.152301	0.267148	0.180406
11	0.123634	0.201778	0.264462	0.202591	0.175305	0.166895
12	0.129239	0.164877	0.161518	0.210169	0.388562	0.239188
13	0.165526	0.225382	0.205194	0.167114	0.229397	0.21988
14	0.157517	0.200934	0.184091	0.175484	0.245564	0.233888
15	0.124883	0.169943	0.208905	0.240994	0.365432	0.260744
16	0.141917	0.243729	0.170936	0.160289	0.308915	0.295827
17	0.108669	0.157099	0.184539	0.195782	0.258701	0.217463
18	0.172547	0.194879	0.140661	0.136071	0.219904	0.276702
19	0.166767	0.189772	0.194895	0.179418	0.160195	0.165666
20	0.228018	0.21085	0.223851	0.180225	0.263632	0.199603
21	0.381367	0.270656	0.3069	0.224255	0.313706	0.289758
22	0.642227	0.622559	0.449651	0.286316	0.273487	0.353465
23	0.596389	0.644164	0.635156	0.515296	0.52329	0.264286
24	0.237484	0.547955	0.576886	0.527484	0.563085	0.572952
25	0.296231	0.343583	0.32292	0.396212	0.573478	0.513413
26	0.386686	0.227447	0.306586	0.337041	0.351029	0.369678
27	0.429457	0.57932	0.450991	0.286039	0.519138	0.489221
28	0.15128	0.239583	0.36572	0.474093	0.489174	0.384164
29	0.140684	0.244179	0.227847	0.185239	0.307265	0.412994
30	0.349623	0.329912	0.219903	0.268654	0.346069	0.195636
31	0.213785	0.402799	0.40365	0.250496	0.354423	0.239902
32	0.398905	0.175069	0.282241	0.368313	0.499888	0.263402
33	0.210275	0.436551	0.430345	0.184794	0.255177	0.300688
34	0.55388	0.498903	0.30813	0.366041	0.359949	0.243389
35	0.127237	0.470472	0.605845	0.479511	0.346718	0.241392
36	0.153614	0.173807	0.192361	0.360788	0.380822	0.370894

Without VSL (Random Seed 35)

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
1	0.09739	0.122911	0.122687	0.123205	0.13195	0.132098
2	0.097497	0.112418	0.114415	0.116787	0.132522	0.134096
3	0.104771	0.128616	0.130759	0.122797	0.127632	0.131892
4	0.113918	0.160284	0.144312	0.148657	0.161829	0.160437
5	0.138287	0.202915	0.181905	0.168705	0.188845	0.189326
6	0.138692	0.20617	0.190219	0.160895	0.207833	0.178093
7	0.11149	0.195565	0.206679	0.186908	0.225801	0.228543
8	0.123308	0.154743	0.138829	0.176217	0.472741	0.294529
9	0.164022	0.163575	0.172302	0.161413	0.187256	0.367522
10	0.132272	0.146005	0.224104	0.201689	0.185186	0.16472
11	0.161025	0.202501	0.143923	0.144878	0.173615	0.162647
12	0.152404	0.158836	0.208146	0.232566	0.267727	0.187302
13	0.112091	0.158791	0.200475	0.16352	0.404637	0.449308
14	0.247693	0.326128	0.167153	0.186504	0.210123	0.215524
15	0.14991	0.186153	0.299941	0.364078	0.220618	0.209982
16	0.182454	0.233138	0.170566	0.203055	0.237466	0.200057
17	0.193789	0.211602	0.312994	0.284911	0.229505	0.192124
18	0.168807	0.294472	0.23787	0.210811	0.558803	0.491416
19	0.213002	0.286212	0.200177	0.211526	0.263866	0.276278
20	0.214874	0.426605	0.326537	0.315425	0.357795	0.258845
21	0.328891	0.349776	0.32625	0.33279	0.341575	0.384065
22	0.59054	0.449074	0.377906	0.347005	0.367921	0.364589
23	0.15921	0.399961	0.669239	0.524297	0.57362	0.358032
24	0.201136	0.308669	0.288627	0.542916	0.699545	0.461025
25	0.609726	0.491179	0.275678	0.23142	0.400681	0.450394
26	0.187128	0.455023	0.583794	0.471508	0.485346	0.374533
27	0.162695	0.359003	0.314216	0.502367	0.506136	0.421979
28	0.155712	0.260167	0.346178	0.358504	0.282484	0.396023
29	0.305859	0.220269	0.232657	0.289902	0.322309	0.338113
30	0.215623	0.42449	0.364706	0.222354	0.309859	0.24676
31	0.28727	0.376017	0.353223	0.389018	0.700487	0.322609
32	0.147892	0.255046	0.32855	0.377846	0.438839	0.5718

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
33	0.177502	0.153314	0.164632	0.208603	0.568322	0.555629
34	0.158095	0.281887	0.222445	0.152771	0.213813	0.266332
35	0.178111	0.169985	0.21648	0.232817	0.281878	0.204009
36	0.412551	0.319057	0.262324	0.184544	0.28598	0.284622

With VSL (Random Seed 35)

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
1	0.09739	0.122911	0.122687	0.123205	0.13195	0.132098
2	0.097497	0.112418	0.114415	0.116787	0.132522	0.134096
3	0.104771	0.128616	0.130759	0.122797	0.127632	0.131892
4	0.113918	0.160284	0.144312	0.148657	0.161829	0.160437
5	0.138287	0.202915	0.181905	0.168705	0.188845	0.189326
6	0.138692	0.20617	0.190219	0.160895	0.207833	0.178093
7	0.11149	0.195565	0.206679	0.186908	0.225801	0.228543
8	0.123308	0.154743	0.138829	0.176217	0.472741	0.294529
9	0.164022	0.163575	0.172302	0.161413	0.187256	0.367522
10	0.132272	0.146005	0.224104	0.201689	0.185186	0.16472
11	0.161025	0.202501	0.143923	0.144878	0.173615	0.162647
12	0.152404	0.158836	0.208146	0.232566	0.267727	0.187302
13	0.112091	0.158791	0.200475	0.16352	0.404637	0.449308
14	0.247693	0.326128	0.167153	0.186504	0.210123	0.215524
15	0.14991	0.186153	0.299941	0.364078	0.220618	0.209982
16	0.182454	0.233138	0.170566	0.203055	0.237466	0.200057
17	0.193789	0.211602	0.312994	0.284911	0.229505	0.192124
18	0.168807	0.294472	0.23787	0.210811	0.558803	0.491416
19	0.213002	0.18685	0.193969	0.201945	0.276001	0.298047
20	0.214718	0.328785	0.22512	0.18271	0.371268	0.204106
21	0.32887	0.289131	0.353872	0.288545	0.462757	0.310684
22	0.602084	0.543239	0.308364	0.303291	0.475341	0.390236
23	0.159194	0.502169	0.611097	0.438039	0.444625	0.239811
24	0.201136	0.308739	0.233439	0.290224	0.57135	0.507737
25	0.609726	0.491179	0.275678	0.23142	0.350577	0.428114

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
26	0.187128	0.423967	0.524305	0.458779	0.489766	0.34562
27	0.162987	0.283359	0.248282	0.320193	0.534884	0.527224
28	0.156277	0.254099	0.282586	0.233335	0.31936	0.3007
29	0.305859	0.21928	0.247536	0.29509	0.258193	0.329429
30	0.215623	0.42449	0.364706	0.220978	0.225701	0.193901
31	0.28727	0.376017	0.353223	0.389018	0.700487	0.322609
32	0.147892	0.255046	0.32855	0.377846	0.438839	0.5718
33	0.177502	0.153314	0.164632	0.208603	0.568322	0.555629
34	0.158095	0.281887	0.222445	0.152771	0.213813	0.266332
35	0.178111	0.169985	0.21648	0.232817	0.281878	0.204009
36	0.412551	0.319057	0.262324	0.184544	0.28598	0.284622

Without VSL (Random Seed 42)

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
1	0.097638	0.12606	0.124482	0.130449	0.128019	0.14531
2	0.108929	0.142038	0.13402	0.125839	0.131402	0.130717
3	0.106703	0.152113	0.155392	0.160991	0.175859	0.17851
4	0.110446	0.142993	0.134057	0.13523	0.153121	0.154176
5	0.16278	0.156442	0.144407	0.147033	0.169752	0.160899
6	0.113966	0.196611	0.204043	0.151522	0.162735	0.175305
7	0.128604	0.176713	0.163559	0.217691	0.225236	0.20517
8	0.124141	0.155168	0.166529	0.163336	0.196324	0.238148
9	0.130516	0.185011	0.166824	0.162375	0.16256	0.148276
10	0.143986	0.14664	0.14821	0.165062	0.184814	0.19177
11	0.180021	0.242525	0.203861	0.16507	0.211624	0.208536
12	0.140488	0.230102	0.250168	0.225227	0.218379	0.209227
13	0.129619	0.222419	0.235407	0.204967	0.341026	0.270465
14	0.116268	0.17245	0.162441	0.168792	0.246335	0.249635
15	0.137539	0.178713	0.145636	0.176342	0.253141	0.151734
16	0.181307	0.185063	0.205822	0.197491	0.253349	0.24163
17	0.123451	0.158245	0.218251	0.162854	0.279859	0.218657
18	0.155125	0.176236	0.150875	0.179871	0.360803	0.239554

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
19	0.136104	0.270241	0.25816	0.18235	0.177213	0.207869
20	0.599181	0.538298	0.334986	0.327939	0.250989	0.234136
21	0.336541	0.471994	0.504018	0.463001	0.371712	0.340677
22	0.213614	0.393414	0.297697	0.277575	0.460661	0.540159
23	0.202264	0.296891	0.269174	0.243441	0.678679	0.475262
24	0.62823	0.471095	0.235007	0.227612	0.251654	0.395415
25	0.378329	0.541837	0.641719	0.46028	0.304713	0.272226
26	0.520883	0.592333	0.43669	0.469492	0.587173	0.345145
27	0.292438	0.520117	0.602202	0.529891	0.43262	0.520745
28	0.342072	0.411622	0.542254	0.46231	0.510618	0.569241
29	0.186317	0.392798	0.418566	0.321318	0.513611	0.389397
30	0.1449	0.180587	0.262835	0.372065	0.322328	0.293925
31	0.26936	0.222742	0.186073	0.178734	0.533667	0.596728
32	0.244692	0.368006	0.325669	0.291406	0.239589	0.237073
33	0.154826	0.293627	0.275662	0.318192	0.251028	0.214041
34	0.143991	0.171011	0.183173	0.327398	0.30642	0.250247
35	0.19156	0.269202	0.219149	0.164943	0.21597	0.257235
36	0.352772	0.441369	0.29003	0.299237	0.288287	0.202791

With VSL (Random Seed 42)

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
1	0.097638	0.12606	0.124482	0.130449	0.128019	0.14531
2	0.108929	0.142038	0.13402	0.125839	0.131402	0.130717
3	0.106703	0.152113	0.155392	0.160991	0.175859	0.17851
4	0.110446	0.142993	0.134057	0.13523	0.153121	0.154176
5	0.16278	0.156442	0.144407	0.147033	0.169752	0.160899
6	0.113966	0.196611	0.204043	0.151522	0.162735	0.175305
7	0.128604	0.176713	0.163559	0.217691	0.225236	0.20517
8	0.124141	0.155168	0.166529	0.163336	0.196324	0.238148
9	0.130516	0.185011	0.166824	0.162375	0.16256	0.148276
10	0.143986	0.14664	0.14821	0.165062	0.184814	0.19177
11	0.180021	0.242525	0.203861	0.16507	0.211624	0.208536

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
12	0.140488	0.230102	0.250168	0.225227	0.218379	0.209227
13	0.129619	0.222419	0.235407	0.204967	0.341026	0.270465
14	0.116268	0.17245	0.162441	0.168792	0.246335	0.249635
15	0.137539	0.178713	0.145636	0.176342	0.253141	0.151734
16	0.181307	0.185063	0.205822	0.197491	0.253349	0.24163
17	0.123451	0.158245	0.218251	0.162854	0.279859	0.218657
18	0.155125	0.176236	0.150875	0.179871	0.360803	0.239554
19	0.136104	0.204825	0.219244	0.165981	0.17692	0.209102
20	0.58495	0.506456	0.205532	0.23636	0.353592	0.25237
21	0.338355	0.451461	0.604637	0.468025	0.276406	0.268073
22	0.213614	0.329789	0.361636	0.403365	0.425524	0.323863
23	0.202264	0.298266	0.255311	0.235695	0.563761	0.252178
24	0.62823	0.471095	0.236628	0.241868	0.335196	0.414872
25	0.378329	0.541837	0.641719	0.46028	0.311313	0.259187
26	0.520869	0.602162	0.457847	0.429054	0.617335	0.402457
27	0.292415	0.448477	0.570246	0.515058	0.395843	0.481608
28	0.342072	0.348334	0.424206	0.42799	0.553457	0.398825
29	0.186317	0.389371	0.388085	0.263245	0.407227	0.384079
30	0.1449	0.180587	0.262835	0.371124	0.342358	0.267534
31	0.26936	0.222742	0.186073	0.178734	0.533667	0.596728
32	0.244692	0.368006	0.325669	0.291406	0.239589	0.237073
33	0.154826	0.293627	0.275662	0.318192	0.251028	0.214041
34	0.143991	0.171011	0.183173	0.327398	0.30642	0.250247
35	0.19156	0.269202	0.219149	0.164943	0.21597	0.257235
36	0.352772	0.441369	0.29003	0.299237	0.288287	0.202791

Without VSL (Random Seed 45)

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
1	0.111993	0.152805	0.146002	0.155592	0.154359	0.143356
2	0.100768	0.120059	0.124339	0.130791	0.141086	0.159531
3	0.101464	0.13759	0.132672	0.140416	0.138737	0.143475
4	0.12604	0.156632	0.138116	0.138385	0.177001	0.15748

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
5	0.151001	0.216076	0.186384	0.158286	0.158268	0.159605
6	0.190196	0.186721	0.192507	0.194049	0.244896	0.183142
7	0.163211	0.222795	0.222994	0.164749	0.233431	0.204589
8	0.139843	0.220385	0.203387	0.240211	0.276627	0.210235
9	0.141375	0.218381	0.166997	0.201921	0.307074	0.203917
10	0.113934	0.183409	0.215816	0.199059	0.175282	0.226803
11	0.127799	0.176977	0.161637	0.201758	0.485549	0.458264
12	0.12678	0.165447	0.166792	0.154486	0.188932	0.190237
13	0.112492	0.162717	0.169027	0.163464	0.195162	0.160002
14	0.158673	0.19549	0.185949	0.184613	0.199328	0.229951
15	0.111431	0.189225	0.252216	0.204884	0.211991	0.197807
16	0.188625	0.19724	0.141191	0.172092	0.28465	0.165454
17	0.515085	0.208138	0.220812	0.20419	0.204511	0.246942
18	0.340656	0.607474	0.571067	0.306811	0.206444	0.211716
19	0.162102	0.283153	0.367956	0.500361	0.615549	0.608549
20	0.175814	0.323896	0.303316	0.303406	0.422362	0.479464
21	0.369364	0.381896	0.269714	0.229067	0.428534	0.321856
22	0.628445	0.574635	0.489824	0.351717	0.453756	0.492404
23	0.243265	0.560267	0.606469	0.525503	0.452715	0.56414
24	0.1942	0.269304	0.298467	0.500064	0.464549	0.403547
25	0.335806	0.363603	0.186363	0.21218	0.218759	0.269557
26	0.600149	0.513267	0.437649	0.473107	0.353471	0.261895
27	0.457841	0.613969	0.739025	0.72814	0.436794	0.41435
28	0.478677	0.619047	0.60258	0.593264	0.620493	0.511839
29	0.309913	0.290991	0.467664	0.493002	0.490457	0.484395
30	0.453503	0.346033	0.330377	0.293115	0.257465	0.345248
31	0.537211	0.62463	0.517575	0.301366	0.356866	0.313626
32	0.247915	0.373137	0.559284	0.567997	0.379987	0.249487
33	0.227148	0.274763	0.256996	0.278851	0.524082	0.541514
34	0.117151	0.198129	0.266119	0.276803	0.318555	0.255704
35	0.51748	0.370729	0.125523	0.159154	0.304999	0.30731
36	0.307984	0.422543	0.470148	0.377792	0.165361	0.259215

With VSL (Random Seed 45)

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
1	0.111993	0.152805	0.146002	0.155592	0.154359	0.143356
2	0.100768	0.120059	0.124339	0.130791	0.141086	0.159531
3	0.101464	0.13759	0.132672	0.140416	0.138737	0.143475
4	0.12604	0.156632	0.138116	0.138385	0.177001	0.15748
5	0.151001	0.216076	0.186384	0.158286	0.158268	0.159605
6	0.190196	0.186721	0.192507	0.194049	0.244896	0.183142
7	0.163211	0.222795	0.222994	0.164749	0.233431	0.204589
8	0.139843	0.220385	0.203387	0.240211	0.276627	0.210235
9	0.141375	0.218381	0.166997	0.201921	0.307074	0.203917
10	0.113934	0.183409	0.215816	0.199059	0.175282	0.226803
11	0.127799	0.176977	0.161637	0.201758	0.485549	0.458264
12	0.12678	0.165447	0.166792	0.154486	0.188932	0.190237
13	0.112492	0.162717	0.169027	0.163464	0.195162	0.160002
14	0.158673	0.19549	0.185949	0.184613	0.199328	0.229951
15	0.111431	0.189225	0.252216	0.204884	0.211991	0.197807
16	0.188625	0.19724	0.141191	0.172092	0.28465	0.165454
17	0.515085	0.208138	0.220812	0.20419	0.204511	0.246942
18	0.340656	0.607474	0.571067	0.306811	0.206444	0.211716
19	0.162235	0.189435	0.357335	0.496001	0.620579	0.608549
20	0.176285	0.200688	0.202737	0.188682	0.319905	0.467235
21	0.369364	0.331789	0.240586	0.215765	0.300826	0.233285
22	0.623159	0.584012	0.46034	0.334028	0.364483	0.219599
23	0.247762	0.581677	0.63353	0.528741	0.482023	0.532346
24	0.1942	0.277816	0.312504	0.51015	0.628751	0.457997
25	0.335806	0.363603	0.186363	0.218036	0.207916	0.45873
26	0.59772	0.486659	0.380298	0.409609	0.257717	0.213006
27	0.458182	0.534082	0.579774	0.476211	0.45523	0.244226
28	0.47781	0.623073	0.495263	0.391073	0.435154	0.460684
29	0.309913	0.283267	0.455637	0.544074	0.509118	0.25593
30	0.453503	0.346033	0.330377	0.279424	0.349858	0.418178
31	0.537211	0.62463	0.517575	0.301366	0.356866	0.312893
32	0.247915	0.373137	0.559284	0.567997	0.379987	0.249487

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
33	0.227148	0.274763	0.256996	0.278851	0.524082	0.541514
34	0.117151	0.198129	0.266119	0.276803	0.318555	0.255704
35	0.51748	0.370729	0.125523	0.159154	0.304999	0.30731
36	0.307984	0.422543	0.470148	0.377792	0.165361	0.259215

Without VSL (Random Seed 47)

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
1	0.100539	0.119627	0.121278	0.131882	0.137647	0.136987
2	0.108453	0.147588	0.143766	0.130598	0.143398	0.153537
3	0.111841	0.161236	0.159678	0.159198	0.195458	0.170795
4	0.117436	0.153197	0.158115	0.16196	0.184958	0.207002
5	0.106096	0.159154	0.162654	0.142852	0.154564	0.159876
6	0.10538	0.14368	0.140434	0.146952	0.212648	0.183319
7	0.13409	0.178616	0.167148	0.163052	0.195781	0.178273
8	0.151724	0.182083	0.176379	0.16401	0.150191	0.154836
9	0.182709	0.224324	0.189871	0.170057	0.201658	0.18809
10	0.118037	0.22156	0.258219	0.245189	0.309072	0.235405
11	0.107158	0.176162	0.1737	0.236753	0.248352	0.208754
12	0.17045	0.158885	0.131691	0.146025	0.21669	0.220226
13	0.313783	0.217155	0.225422	0.19805	0.172557	0.173125
14	0.287572	0.422406	0.35697	0.177561	0.521988	0.360733
15	0.12512	0.193919	0.197117	0.348392	0.470394	0.265302
16	0.125189	0.19048	0.175958	0.196492	0.289298	0.441296
17	0.203708	0.175048	0.170879	0.158712	0.262866	0.291819
18	0.23335	0.377372	0.263102	0.151401	0.200507	0.225021
19	0.413383	0.260721	0.304776	0.382302	0.219806	0.149634
20	0.526191	0.609549	0.499367	0.344138	0.353515	0.251594
21	0.402765	0.571522	0.616851	0.549885	0.466245	0.446744
22	0.484813	0.446166	0.486371	0.484743	0.52965	0.506676
23	0.290846	0.536059	0.52072	0.394599	0.479612	0.453791
24	0.53483	0.484176	0.372885	0.440498	0.377342	0.301257
25	0.187009	0.460935	0.555466	0.4905	0.579674	0.351957

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
26	0.242868	0.319406	0.311196	0.343712	0.497762	0.553042
27	0.384176	0.415476	0.381453	0.354649	0.591756	0.376361
28	0.277213	0.360177	0.365959	0.456075	0.357356	0.503441
29	0.55875	0.606974	0.343917	0.359654	0.614943	0.386343
30	0.267686	0.404768	0.590603	0.594643	0.600442	0.504339
31	0.154783	0.224125	0.292437	0.492899	0.674936	0.487609
32	0.392702	0.444851	0.24335	0.224512	0.270031	0.476165
33	0.15186	0.192916	0.334717	0.287573	0.249057	0.213806
34	0.186688	0.169759	0.19493	0.202524	0.312822	0.242579
35	0.115765	0.239606	0.234523	0.153707	0.241197	0.217163
36	0.122739	0.167079	0.170356	0.254277	0.57055	0.195212

With VSL (Random Seed 47)

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
1	0.100539	0.119627	0.121278	0.131882	0.137647	0.136987
2	0.108453	0.147588	0.143766	0.130598	0.143398	0.153537
3	0.111841	0.161236	0.159678	0.159198	0.195458	0.170795
4	0.117436	0.153197	0.158115	0.16196	0.184958	0.207002
5	0.106096	0.159154	0.162654	0.142852	0.154564	0.159876
6	0.10538	0.14368	0.140434	0.146952	0.212648	0.183319
7	0.13409	0.178616	0.167148	0.163052	0.195781	0.178273
8	0.151724	0.182083	0.176379	0.16401	0.150191	0.154836
9	0.182709	0.224324	0.189871	0.170057	0.201658	0.18809
10	0.118037	0.22156	0.258219	0.245189	0.309072	0.235405
11	0.107158	0.176162	0.1737	0.236753	0.248352	0.208754
12	0.17045	0.158885	0.131691	0.146025	0.21669	0.220226
13	0.313783	0.217155	0.225422	0.19805	0.172557	0.173125
14	0.287572	0.422406	0.35697	0.177561	0.521988	0.360733
15	0.12512	0.193919	0.197117	0.348392	0.470394	0.265302
16	0.125189	0.19048	0.175958	0.196492	0.289298	0.441296
17	0.203708	0.175048	0.170879	0.158712	0.262866	0.291819
18	0.23335	0.377372	0.263102	0.151401	0.200507	0.225021

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
19	0.413383	0.198094	0.352442	0.378296	0.189838	0.147442
20	0.537121	0.639479	0.427646	0.209045	0.257064	0.234824
21	0.402765	0.563405	0.50731	0.464677	0.358119	0.247559
22	0.487151	0.401318	0.474559	0.43936	0.399598	0.371082
23	0.290846	0.557942	0.550221	0.379	0.428907	0.280163
24	0.53483	0.484176	0.404082	0.54215	0.400916	0.290361
25	0.187009	0.460935	0.555466	0.494519	0.601325	0.48765
26	0.242868	0.222596	0.248966	0.256315	0.37928	0.602424
27	0.384176	0.450531	0.387143	0.250556	0.562494	0.218721
28	0.277213	0.328492	0.280106	0.37987	0.353985	0.494748
29	0.55875	0.613275	0.363815	0.322092	0.603115	0.471783
30	0.267686	0.358275	0.60063	0.576775	0.432469	0.421856
31	0.154783	0.224125	0.295549	0.428684	0.625891	0.545229
32	0.392702	0.444851	0.24335	0.232127	0.29112	0.424642
33	0.15186	0.192916	0.334717	0.287573	0.249057	0.213826
34	0.186688	0.169759	0.19493	0.202524	0.312822	0.242579
35	0.115765	0.239606	0.234523	0.153707	0.241197	0.217163
36	0.122739	0.167079	0.170356	0.254277	0.57055	0.195212

Without VSL (Random Seed 50)

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
1	0.103515	0.119168	0.123496	0.125574	0.142846	0.14088
2	0.099195	0.124143	0.123005	0.119343	0.122878	0.124277
3	0.103503	0.127553	0.129926	0.128225	0.146287	0.148364
4	0.113346	0.130001	0.127352	0.13502	0.152812	0.137122
5	0.12046	0.151898	0.156364	0.129364	0.137816	0.149466
6	0.147246	0.152953	0.157122	0.152312	0.172665	0.159552
7	0.146051	0.209777	0.196441	0.171796	0.192329	0.185332
8	0.215339	0.269312	0.23572	0.195689	0.470783	0.280692
9	0.213972	0.379504	0.239852	0.293246	0.267685	0.312577
10	0.179227	0.267256	0.299318	0.346745	0.242294	0.28126
11	0.173354	0.235133	0.22554	0.180477	0.329014	0.290877

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
12	0.130853	0.138713	0.183289	0.225801	0.370568	0.218226
13	0.133791	0.196197	0.162727	0.142879	0.199947	0.224341
14	0.135387	0.197633	0.246061	0.195136	0.264148	0.17582
15	0.235643	0.238876	0.189281	0.17612	0.26297	0.255328
16	0.151141	0.293985	0.301643	0.21457	0.216767	0.220673
17	0.130866	0.158959	0.174766	0.337847	0.386622	0.265816
18	0.185399	0.214228	0.208019	0.189727	0.232504	0.201494
19	0.160083	0.257324	0.257462	0.203083	0.33443	0.29295
20	0.475033	0.434894	0.352052	0.295125	0.334326	0.305311
21	0.641429	0.60353	0.509732	0.310235	0.30299	0.249316
22	0.470541	0.646635	0.620563	0.494584	0.543988	0.423515
23	0.276073	0.375228	0.511038	0.47329	0.584723	0.59269
24	0.244261	0.244532	0.292562	0.225404	0.420374	0.576607
25	0.222122	0.275025	0.266653	0.24625	0.336275	0.254474
26	0.191969	0.420521	0.404496	0.312689	0.406505	0.320334
27	0.33253	0.287892	0.331067	0.408925	0.497492	0.391242
28	0.637974	0.573767	0.421471	0.400318	0.276233	0.455814
29	0.57453	0.592109	0.66871	0.502677	0.452028	0.34531
30	0.172639	0.434672	0.487452	0.561803	0.6167	0.569669
31	0.169635	0.320053	0.215974	0.275133	0.638031	0.602997
32	0.219897	0.259368	0.213285	0.321175	0.264166	0.322776
33	0.214581	0.317161	0.243254	0.240031	0.215228	0.238267
34	0.115171	0.179092	0.238938	0.224681	0.254352	0.236778
35	0.180089	0.162311	0.150437	0.16528	0.213595	0.28019
36	0.297189	0.423247	0.254526	0.170396	0.222486	0.183207

With VSL (Random Seed 50)

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
1	0.103515	0.119168	0.123496	0.125574	0.142846	0.14088
2	0.099195	0.124143	0.123005	0.119343	0.122878	0.124277
3	0.103503	0.127553	0.129926	0.128225	0.146287	0.148364
4	0.113346	0.130001	0.127352	0.13502	0.152812	0.137122

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
5	0.12046	0.151898	0.156364	0.129364	0.137816	0.149466
6	0.147246	0.152953	0.157122	0.152312	0.172665	0.159552
7	0.146051	0.209777	0.196441	0.171796	0.192329	0.185332
8	0.215339	0.269312	0.23572	0.195689	0.470783	0.280692
9	0.213972	0.379504	0.239852	0.293246	0.267685	0.312577
10	0.179227	0.267256	0.299318	0.346745	0.242294	0.28126
11	0.173354	0.235133	0.22554	0.180477	0.329014	0.290877
12	0.130853	0.138713	0.183289	0.225801	0.370568	0.218226
13	0.133791	0.196197	0.162727	0.142879	0.199947	0.224341
14	0.135387	0.197633	0.246061	0.195136	0.264148	0.17582
15	0.235643	0.238876	0.189281	0.17612	0.26297	0.255328
16	0.151141	0.293985	0.301643	0.21457	0.216767	0.220673
17	0.130866	0.158959	0.174766	0.337847	0.386622	0.265816
18	0.185399	0.214228	0.208019	0.189727	0.232504	0.201494
19	0.160083	0.173022	0.190088	0.235641	0.322143	0.261075
20	0.475153	0.454554	0.246016	0.176408	0.212191	0.251713
21	0.644306	0.6011	0.527106	0.271823	0.286659	0.174044
22	0.484418	0.63133	0.611163	0.47375	0.427688	0.398684
23	0.276073	0.37698	0.496915	0.538806	0.5102	0.353211
24	0.244261	0.244532	0.292562	0.254516	0.373937	0.439613
25	0.222122	0.275025	0.266653	0.24625	0.336275	0.266121
26	0.190755	0.36734	0.379717	0.277176	0.344486	0.252792
27	0.335266	0.244891	0.244296	0.321141	0.3063	0.279691
28	0.633933	0.578527	0.383982	0.333097	0.288082	0.260586
29	0.577103	0.656616	0.653955	0.575164	0.337549	0.230192
30	0.168461	0.486433	0.616826	0.538136	0.669001	0.510634
31	0.169635	0.320053	0.215206	0.331865	0.602367	0.559195
32	0.219897	0.259368	0.213285	0.321175	0.264166	0.253712
33	0.214581	0.317161	0.243254	0.240031	0.215228	0.238267
34	0.115171	0.179092	0.238938	0.224681	0.254352	0.236778
35	0.180089	0.162311	0.150437	0.16528	0.213595	0.28019
36	0.297189	0.423247	0.254526	0.170396	0.222486	0.183207

Without VSL (Random Seed 55)

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
1	0.101147	0.117292	0.118675	0.117058	0.127347	0.138098
2	0.105606	0.146367	0.138438	0.140606	0.149675	0.144608
3	0.104384	0.140005	0.137374	0.131174	0.144663	0.142291
4	0.111572	0.145268	0.150726	0.1575	0.15185	0.162298
5	0.198697	0.192307	0.186008	0.146744	0.168124	0.174169
6	0.114885	0.1862	0.247868	0.193412	0.223407	0.204213
7	0.114585	0.157718	0.162585	0.163149	0.242353	0.249227
8	0.153354	0.20092	0.162445	0.145822	0.180755	0.193166
9	0.259821	0.266351	0.230369	0.210407	0.212528	0.182859
10	0.170827	0.254711	0.275977	0.269977	0.397603	0.244652
11	0.129553	0.173381	0.228191	0.253385	0.51757	0.323693
12	0.181329	0.227204	0.194146	0.173133	0.300719	0.222109
13	0.142292	0.168046	0.189436	0.239577	0.203167	0.232891
14	0.251435	0.239774	0.188725	0.170736	0.276653	0.241315
15	0.123589	0.191126	0.282532	0.261644	0.246518	0.206488
16	0.117807	0.147105	0.152475	0.18487	0.486724	0.588294
17	0.374271	0.194065	0.161698	0.15986	0.228897	0.219989
18	0.125624	0.381336	0.436706	0.311752	0.210113	0.194873
19	0.164424	0.292855	0.178002	0.297642	0.275945	0.183045
20	0.582441	0.52669	0.348842	0.314922	0.255584	0.269569
21	0.45819	0.640487	0.593058	0.301985	0.446462	0.28938
22	0.517295	0.504528	0.53035	0.590042	0.543191	0.456435
23	0.166358	0.282368	0.509895	0.47869	0.51411	0.525155
24	0.600111	0.544538	0.27334	0.270397	0.634556	0.621679
25	0.488258	0.582925	0.629741	0.475611	0.397131	0.307237
26	0.358364	0.490611	0.468119	0.46981	0.581985	0.549431
27	0.504428	0.44051	0.407762	0.512392	0.432641	0.346363
28	0.434672	0.575561	0.57972	0.448126	0.52531	0.428684
29	0.24358	0.347026	0.507416	0.44243	0.352792	0.416988
30	0.212674	0.31219	0.262901	0.264164	0.275194	0.223338
31	0.400936	0.278325	0.248023	0.260976	0.460641	0.227902
32	0.617902	0.642151	0.48622	0.239529	0.385269	0.337155

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
33	0.131691	0.34902	0.628734	0.589319	0.260591	0.202052
34	0.259337	0.229535	0.223123	0.371519	0.484453	0.404545
35	0.120026	0.242837	0.180567	0.180044	0.22514	0.256839
36	0.163022	0.226379	0.167576	0.168508	0.289331	0.272806

With VSL (Random Seed 55)

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
1	0.101147	0.117292	0.118675	0.117058	0.127347	0.138098
2	0.105606	0.146367	0.138438	0.140606	0.149675	0.144608
3	0.104384	0.140005	0.137374	0.131174	0.144663	0.142291
4	0.111572	0.145268	0.150726	0.1575	0.15185	0.162298
5	0.198697	0.192307	0.186008	0.146744	0.168124	0.174169
6	0.114885	0.1862	0.247868	0.193412	0.223407	0.204213
7	0.114585	0.157718	0.162585	0.163149	0.242353	0.249227
8	0.153354	0.20092	0.162445	0.145822	0.180755	0.193166
9	0.259821	0.266351	0.230369	0.210407	0.212528	0.182859
10	0.170827	0.254711	0.275977	0.269977	0.397603	0.244652
11	0.129553	0.173381	0.228191	0.253385	0.51757	0.323693
12	0.181329	0.227204	0.194146	0.173133	0.300719	0.222109
13	0.142292	0.168046	0.189436	0.239577	0.203167	0.232891
14	0.251435	0.239774	0.188725	0.170736	0.276653	0.241315
15	0.123589	0.191126	0.282532	0.261644	0.246518	0.206488
16	0.117807	0.147105	0.152475	0.18487	0.486724	0.588294
17	0.374271	0.194065	0.161698	0.15986	0.228897	0.219989
18	0.125624	0.381336	0.436706	0.311752	0.210113	0.194873
19	0.164269	0.180962	0.182911	0.293148	0.27665	0.206548
20	0.582656	0.45781	0.257261	0.208018	0.200428	0.224454
21	0.456767	0.614951	0.598157	0.377952	0.450736	0.197536
22	0.517295	0.47585	0.452523	0.520331	0.344461	0.375411
23	0.166358	0.297697	0.518982	0.488043	0.366104	0.440537
24	0.600111	0.544538	0.27334	0.246289	0.609918	0.624819
25	0.488258	0.582925	0.629741	0.475611	0.397131	0.334124

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
26	0.358521	0.471999	0.483866	0.400676	0.526675	0.544648
27	0.504172	0.38015	0.410676	0.469418	0.556563	0.467866
28	0.434856	0.544751	0.557661	0.400895	0.402174	0.352362
29	0.24358	0.361367	0.454271	0.38592	0.421178	0.375836
30	0.212674	0.31219	0.262901	0.267782	0.29827	0.266855
31	0.400936	0.278325	0.248023	0.260976	0.460641	0.248218
32	0.617902	0.642151	0.48622	0.239529	0.385269	0.337155
33	0.131691	0.34902	0.628734	0.589319	0.260591	0.202052
34	0.259337	0.229535	0.223123	0.371519	0.484453	0.404545
35	0.120026	0.242837	0.180567	0.180044	0.22514	0.256839
36	0.163022	0.226379	0.167576	0.168508	0.289331	0.272806

Without VSL (Random Seed 67)

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
1	0.113193	0.153261	0.156049	0.159171	0.166229	0.176482
2	0.111209	0.144356	0.130905	0.12943	0.139465	0.134482
3	0.107642	0.145409	0.149238	0.147558	0.155703	0.150774
4	0.113182	0.142497	0.140071	0.140857	0.145606	0.148839
5	0.124222	0.170056	0.179027	0.162717	0.199256	0.190263
6	0.112658	0.16781	0.169603	0.17381	0.252112	0.22104
7	0.119321	0.130908	0.140929	0.143984	0.176106	0.230301
8	0.227095	0.271376	0.160683	0.144487	0.152658	0.145449
9	0.132206	0.186046	0.329594	0.296524	0.201731	0.174253
10	0.165363	0.254515	0.182533	0.157695	0.257909	0.270448
11	0.123842	0.152306	0.203963	0.21122	0.240013	0.215568
12	0.117916	0.171216	0.164202	0.147414	0.176084	0.197618
13	0.130544	0.163973	0.170267	0.179343	0.220383	0.214049
14	0.125619	0.190761	0.189422	0.164072	0.173743	0.173126
15	0.17114	0.172439	0.165387	0.178689	0.222302	0.191035
16	0.41466	0.24403	0.280197	0.20252	0.201796	0.194798
17	0.171836	0.522878	0.453575	0.207882	0.282151	0.227219
18	0.122462	0.16895	0.259131	0.395063	0.345374	0.336557

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
19	0.37004	0.27339	0.160472	0.166473	0.185422	0.235648
20	0.542171	0.668473	0.498699	0.347793	0.320862	0.239353
21	0.28581	0.546442	0.558211	0.536386	0.518583	0.375032
22	0.348983	0.212774	0.281647	0.344755	0.664796	0.488302
23	0.569886	0.596183	0.430593	0.213335	0.29888	0.371134
24	0.141649	0.373074	0.618	0.497856	0.628993	0.310364
25	0.620175	0.539525	0.240033	0.331852	0.561849	0.511094
26	0.438389	0.548055	0.637924	0.504013	0.376071	0.328102
27	0.239388	0.45862	0.450104	0.484733	0.593346	0.560034
28	0.520929	0.273589	0.455254	0.534974	0.3847	0.351239
29	0.382411	0.623401	0.546325	0.291683	0.297232	0.366204
30	0.174888	0.288979	0.458908	0.49335	0.452413	0.331512
31	0.217981	0.191306	0.211513	0.237758	0.375481	0.334529
32	0.415824	0.54674	0.284636	0.215884	0.485473	0.446823
33	0.539332	0.481611	0.493919	0.481794	0.434265	0.232788
34	0.190356	0.385729	0.558152	0.462726	0.319174	0.343886
35	0.141614	0.217575	0.240982	0.318403	0.493813	0.414647
36	0.276102	0.376063	0.192323	0.214689	0.280501	0.233624

With VSL (Random Seed 67)

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
1	0.113193	0.153261	0.156049	0.159171	0.166229	0.176482
2	0.111209	0.144356	0.130905	0.12943	0.139465	0.134482
3	0.107642	0.145409	0.149238	0.147558	0.155703	0.150774
4	0.113182	0.142497	0.140071	0.140857	0.145606	0.148839
5	0.124222	0.170056	0.179027	0.162717	0.199256	0.190263
6	0.112658	0.16781	0.169603	0.17381	0.252112	0.22104
7	0.119321	0.130908	0.140929	0.143984	0.176106	0.230301
8	0.227095	0.271376	0.160683	0.144487	0.152658	0.145449
9	0.132206	0.186046	0.329594	0.296524	0.201731	0.174253
10	0.165363	0.254515	0.182533	0.157695	0.257909	0.270448
11	0.123842	0.152306	0.203963	0.21122	0.240013	0.215568

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
12	0.117916	0.171216	0.164202	0.147414	0.176084	0.197618
13	0.130544	0.163973	0.170267	0.179343	0.220383	0.214049
14	0.125619	0.190761	0.189422	0.164072	0.173743	0.173126
15	0.17114	0.172439	0.165387	0.178689	0.222302	0.191035
16	0.41466	0.24403	0.280197	0.20252	0.201796	0.194798
17	0.171836	0.522878	0.453575	0.207882	0.282151	0.227219
18	0.122462	0.16895	0.259131	0.395063	0.345374	0.336557
19	0.37004	0.184247	0.165014	0.170559	0.186924	0.230638
20	0.540217	0.543462	0.422229	0.265305	0.252934	0.189224
21	0.28581	0.524547	0.515775	0.516182	0.419021	0.248725
22	0.349046	0.173699	0.248625	0.372195	0.62489	0.52793
23	0.568769	0.621009	0.385669	0.193555	0.343963	0.306836
24	0.141799	0.363543	0.59684	0.522211	0.635227	0.322072
25	0.620175	0.539525	0.240033	0.268244	0.568152	0.531547
26	0.438389	0.557286	0.622134	0.451136	0.306854	0.28411
27	0.239371	0.529825	0.451875	0.364856	0.489349	0.436832
28	0.520929	0.257192	0.36654	0.508434	0.423029	0.254255
29	0.382452	0.611086	0.507012	0.306238	0.411109	0.339526
30	0.174888	0.289997	0.432398	0.502361	0.433016	0.238314
31	0.217981	0.191306	0.211513	0.237758	0.379797	0.362537
32	0.415824	0.54674	0.284636	0.215884	0.485473	0.446823
33	0.539332	0.481611	0.493919	0.481794	0.434265	0.232788
34	0.190356	0.385729	0.558152	0.462726	0.319174	0.343886
35	0.141614	0.217575	0.240982	0.318403	0.493813	0.414647
36	0.276102	0.376063	0.192323	0.214689	0.280501	0.233624

Without VSL (Random Seed 77)

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
1	0.101986	0.127223	0.134307	0.133212	0.157408	0.176141
2	0.101756	0.128065	0.123263	0.126381	0.132371	0.127352
3	0.104872	0.14427	0.144798	0.148705	0.164923	0.159627
4	0.105705	0.131122	0.133769	0.12826	0.1522	0.159015

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
5	0.115327	0.154352	0.12762	0.130829	0.149645	0.148929
6	0.109457	0.145932	0.15742	0.155349	0.164678	0.181487
7	0.120685	0.148807	0.14188	0.13052	0.154364	0.149688
8	0.129013	0.186482	0.14841	0.145367	0.184008	0.155472
9	0.138233	0.18841	0.218265	0.208199	0.22148	0.206188
10	0.155132	0.174114	0.203627	0.189609	0.301568	0.303713
11	0.230004	0.224399	0.231318	0.172036	0.192972	0.178215
12	0.143554	0.201578	0.273719	0.260438	0.313674	0.20693
13	0.171749	0.240054	0.194254	0.181753	0.202683	0.236469
14	0.194119	0.290728	0.201831	0.227977	0.196032	0.180834
15	0.125094	0.212488	0.206555	0.226922	0.263416	0.214998
16	0.218677	0.197803	0.197924	0.205912	0.311211	0.353909
17	0.120202	0.272817	0.281731	0.279718	0.293853	0.275955
18	0.131145	0.17808	0.172808	0.2102	0.306133	0.315497
19	0.121199	0.244085	0.197813	0.179156	0.165897	0.186486
20	0.138473	0.313418	0.294787	0.285926	0.321602	0.251942
21	0.130514	0.252745	0.263389	0.236761	0.339927	0.261885
22	0.440725	0.285936	0.214324	0.223525	0.317229	0.349477
23	0.382587	0.51984	0.481414	0.250326	0.257115	0.211325
24	0.164701	0.271354	0.394288	0.444216	0.344262	0.20869
25	0.572814	0.454722	0.245725	0.234916	0.264206	0.406328
26	0.65532	0.609537	0.608859	0.49481	0.347279	0.315555
27	0.216003	0.632579	0.615371	0.610691	0.500863	0.625687
28	0.344732	0.256098	0.38137	0.550093	0.475258	0.551276
29	0.205675	0.515271	0.478488	0.279617	0.307363	0.416118
30	0.561268	0.462927	0.269371	0.502945	0.342637	0.177684
31	0.208733	0.493296	0.600331	0.416504	0.380695	0.308514
32	0.433385	0.286024	0.27114	0.374403	0.576354	0.366282
33	0.14306	0.46413	0.445275	0.360878	0.411102	0.468593
34	0.150407	0.208308	0.244266	0.277651	0.430557	0.376032
35	0.121086	0.17914	0.206683	0.210089	0.243371	0.312555
36	0.24781	0.288199	0.168426	0.163043	0.284732	0.23203

With VSL (Random Seed 77)

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
1	0.101986	0.127223	0.134307	0.133212	0.157408	0.176141
2	0.101756	0.128065	0.123263	0.126381	0.132371	0.127352
3	0.104872	0.14427	0.144798	0.148705	0.164923	0.159627
4	0.105705	0.131122	0.133769	0.12826	0.1522	0.159015
5	0.115327	0.154352	0.12762	0.130829	0.149645	0.148929
6	0.109457	0.145932	0.15742	0.155349	0.164678	0.181487
7	0.120685	0.148807	0.14188	0.13052	0.154364	0.149688
8	0.129013	0.186482	0.14841	0.145367	0.184008	0.155472
9	0.138233	0.18841	0.218265	0.208199	0.22148	0.206188
10	0.155132	0.174114	0.203627	0.189609	0.301568	0.303713
11	0.230004	0.224399	0.231318	0.172036	0.192972	0.178215
12	0.143554	0.201578	0.273719	0.260438	0.313674	0.20693
13	0.171749	0.240054	0.194254	0.181753	0.202683	0.236469
14	0.194119	0.290728	0.201831	0.227977	0.196032	0.180834
15	0.125094	0.212488	0.206555	0.226922	0.263416	0.214998
16	0.218677	0.197803	0.197924	0.205912	0.311211	0.353909
17	0.120202	0.272817	0.281731	0.279718	0.293853	0.275955
18	0.131145	0.17808	0.172808	0.2102	0.306133	0.315497
19	0.121199	0.160394	0.216145	0.194712	0.171292	0.184378
20	0.138473	0.17326	0.169769	0.159784	0.316585	0.276312
21	0.130514	0.227012	0.218025	0.188648	0.223765	0.228041
22	0.441062	0.250355	0.165892	0.181773	0.250447	0.297689
23	0.382587	0.517675	0.502082	0.242471	0.247034	0.176687
24	0.164701	0.271354	0.382779	0.462544	0.41057	0.403769
25	0.572814	0.454722	0.245725	0.234916	0.265702	0.248183
26	0.656432	0.631167	0.613662	0.447199	0.282089	0.252006
27	0.212303	0.598111	0.649703	0.629171	0.323427	0.293901
28	0.344732	0.214806	0.261017	0.401149	0.533878	0.460452
29	0.20971	0.475012	0.435181	0.193998	0.273872	0.280566
30	0.561268	0.462893	0.314299	0.427795	0.449026	0.222931
31	0.208733	0.493296	0.600331	0.416504	0.3499	0.406229
32	0.433385	0.286024	0.27114	0.374403	0.576354	0.366282

Interval	Crash Risk U1	Crash Risk U2	Crash Risk U3	Crash Risk U4	Crash Risk D1	Crash Risk D2
33	0.14306	0.46413	0.445275	0.360878	0.411102	0.468593
34	0.150407	0.208308	0.244266	0.277651	0.430557	0.376032
35	0.121086	0.17914	0.206683	0.210089	0.243371	0.312555
36	0.24781	0.288199	0.168426	0.163043	0.284732	0.23203